Transmission-Line Essentials for Digital Electronics

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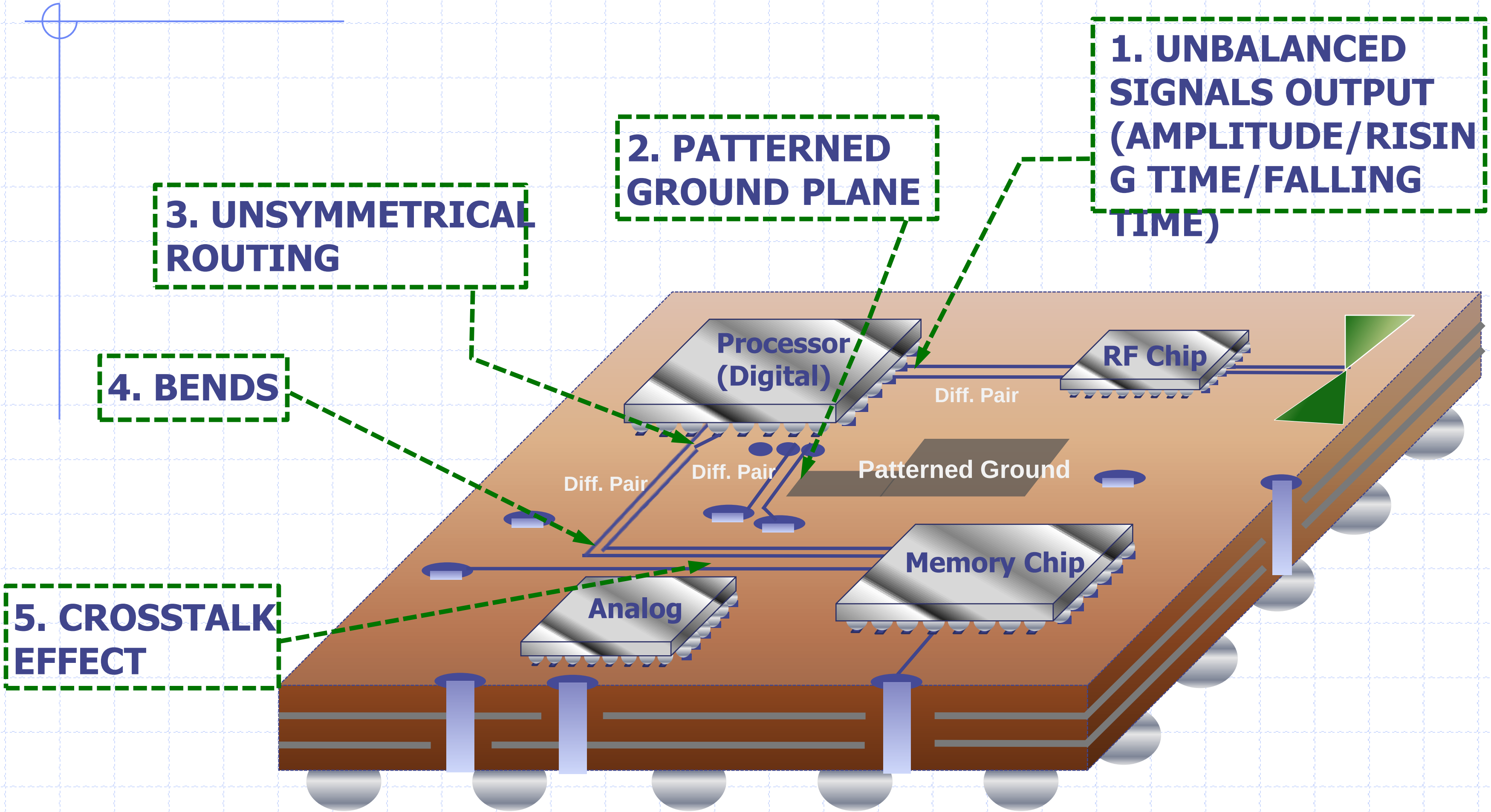
Introduction

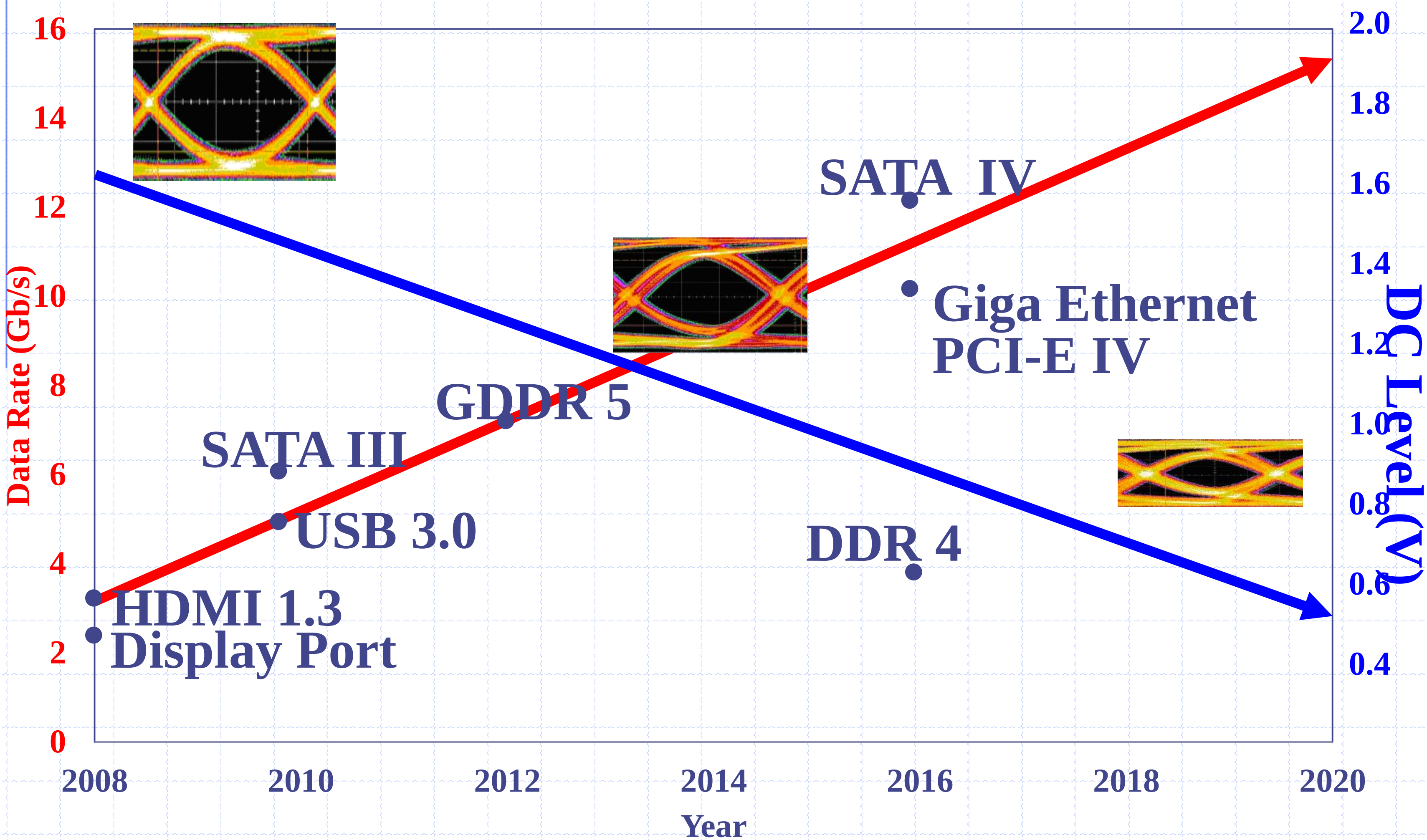
- 6.1 Transmission Line
- 6.2 Line Terminated by Resistive Load
- 6.3 Transmission-Line Discontinuity
- 6.4 Lines with Reactive Terminations and Discontinuities
- 6.5 Lines with Initial Conditions
- 6.6 Interconnections between Logic Gates
- 6.7 Crosstalk on Transmission Lines

Transmission lines



Trend of Differential Signaling

Data: ITRS and IBM



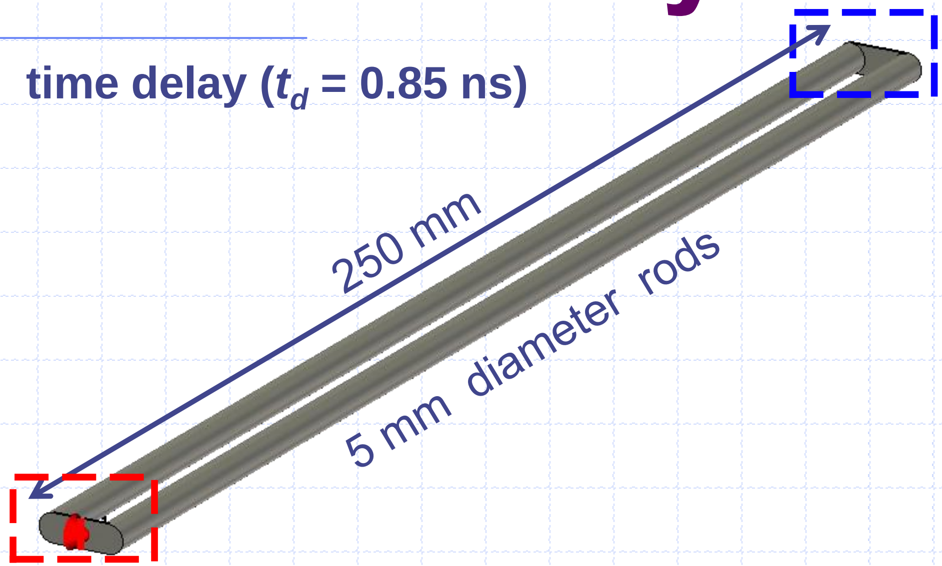
Two Criterions for Transmission Lines

- Two conductors in homogeneous medium.
- The conductors are electrically long.

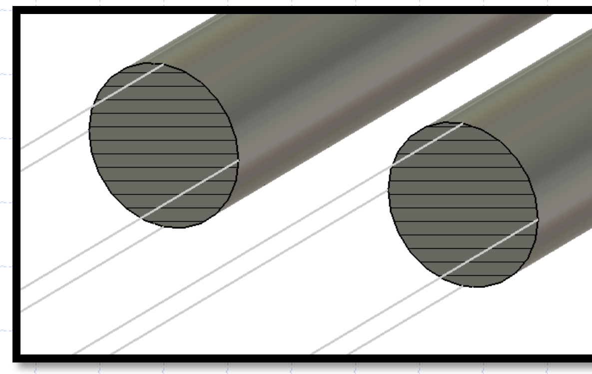
  Good engineering approximation: $t_r < 6t_d$

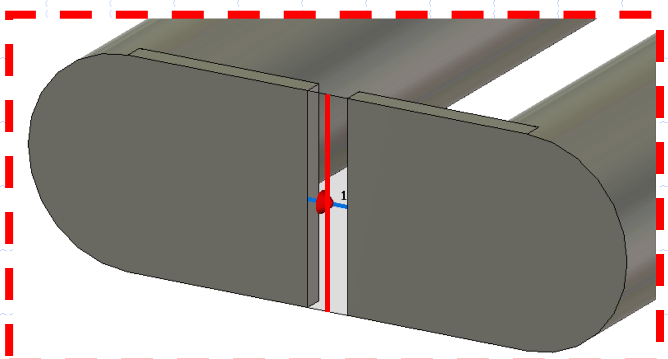
Transmission-Line Geometry

time delay (t_d = 0.85 ns)

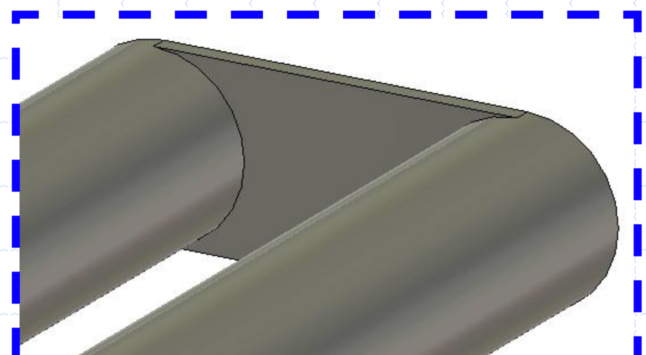


Cross sectional view



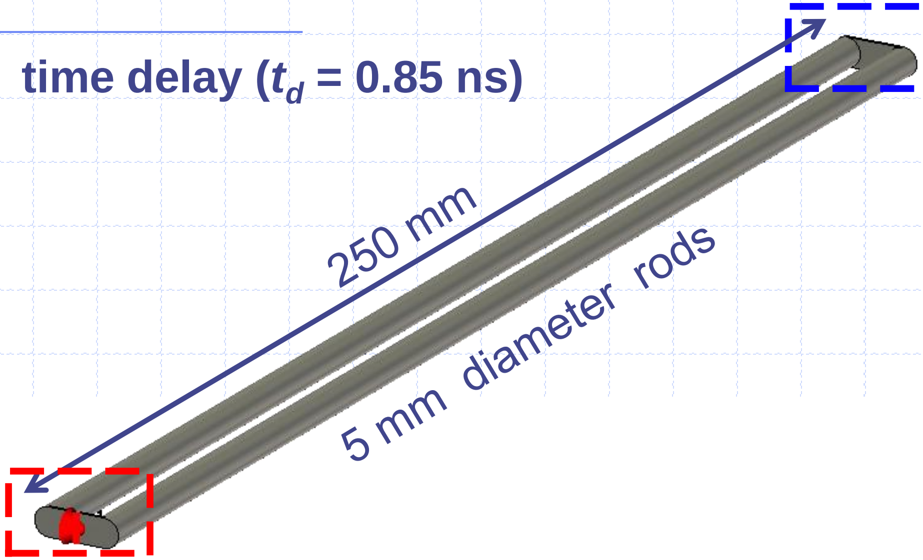


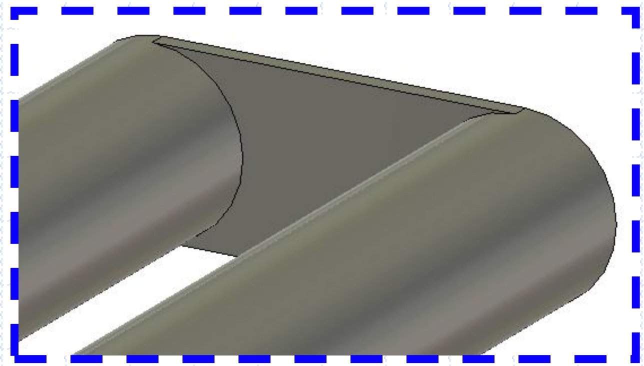
Discrete edge port of 50 Ω



Shorted at the far end

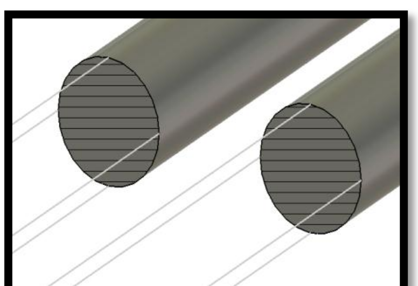
Transmission-Line Surface Current

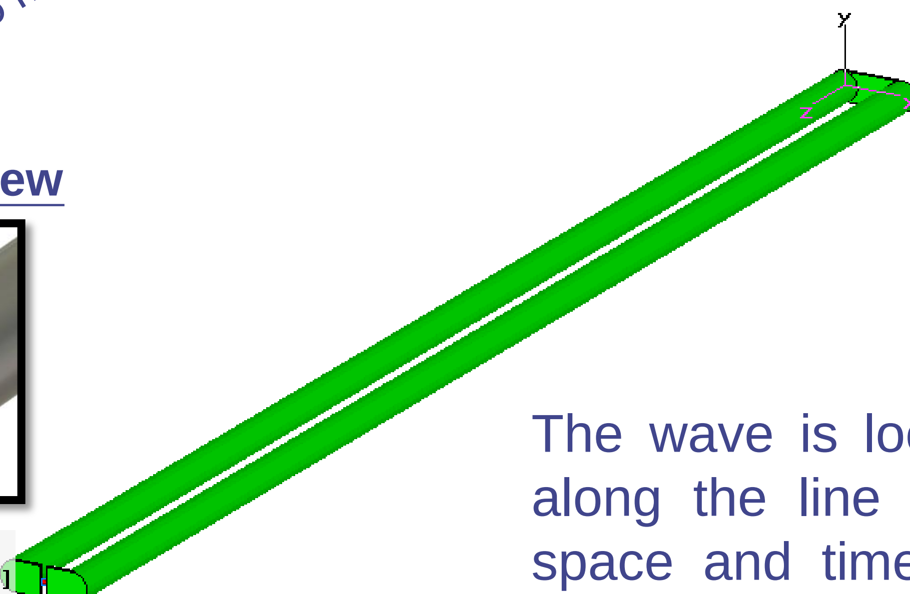


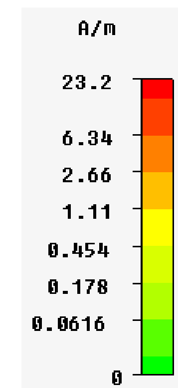


Shorted at the far end

Cross sectional view



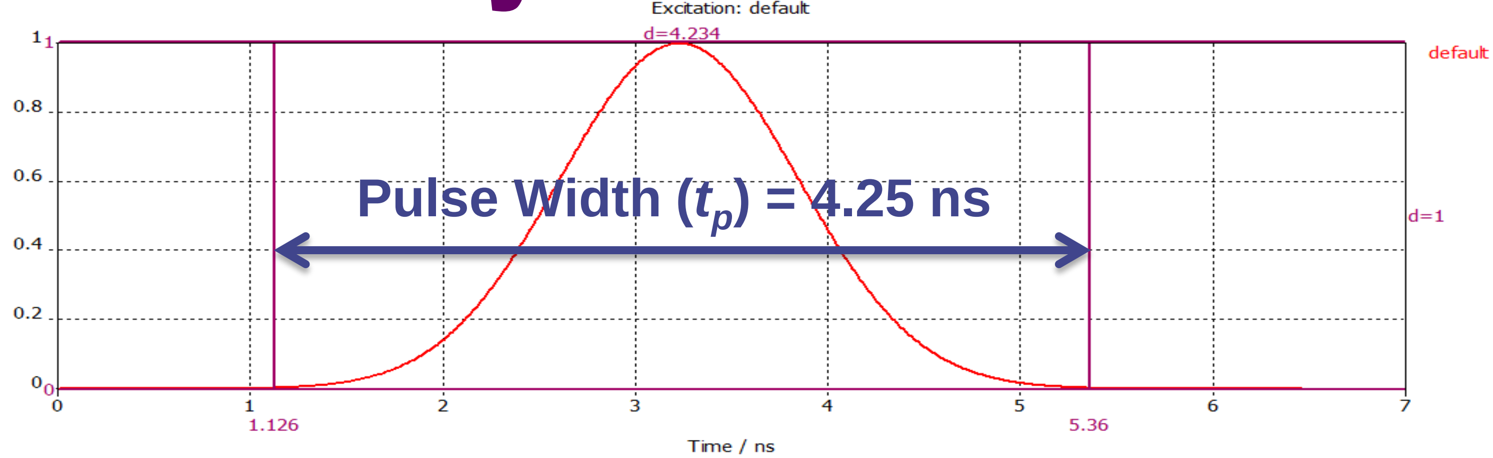




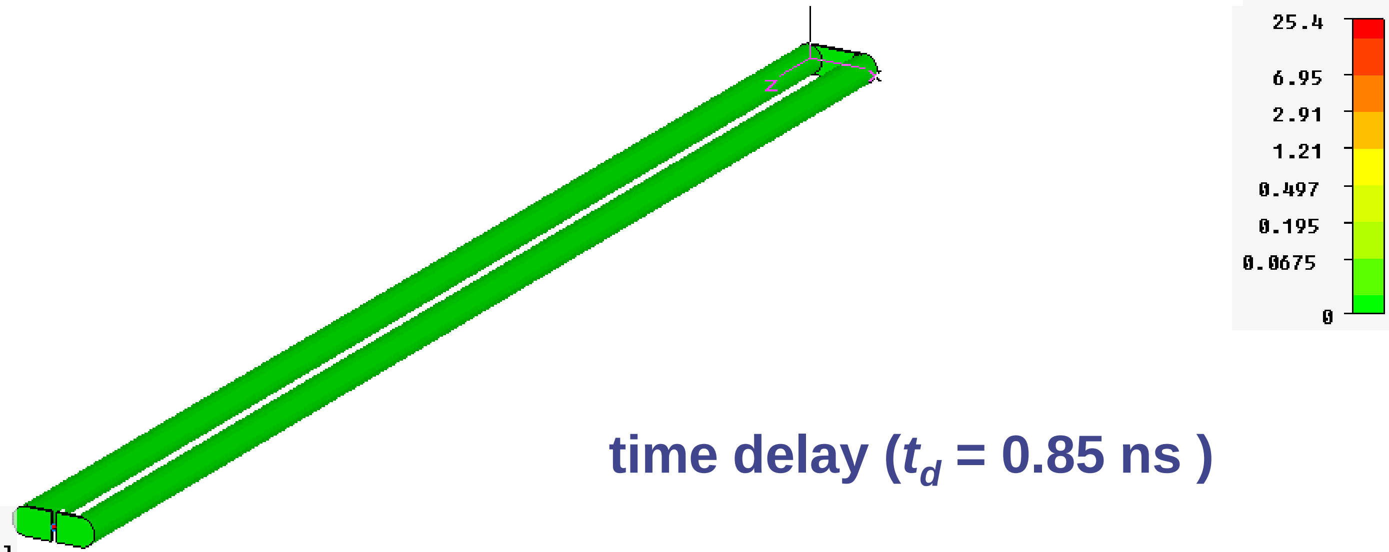
Type	Surface Current (peak)
Monitor	h-field (t=0..end(1e-9)) [1]
Component	Abs
Maximum-3d	23.2196 A/m at 0 / 2.5 / 0.5
Sample	1 / 61
Time	0

The wave is localized in a length along the line and moves out in space and time without changing shape

Electrically Short Twin Lead



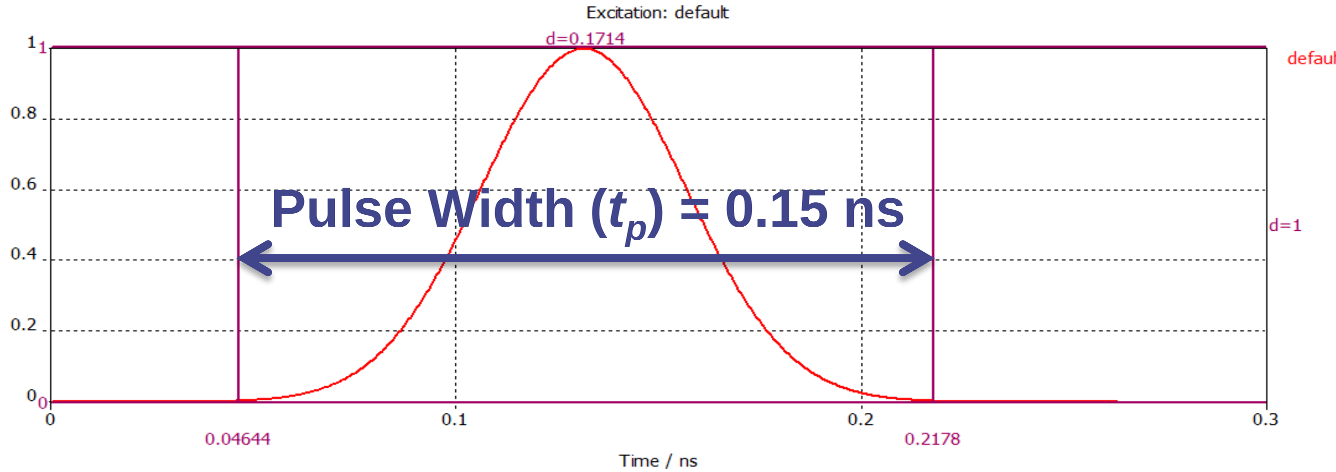
$$@t_p = 5t_d$$



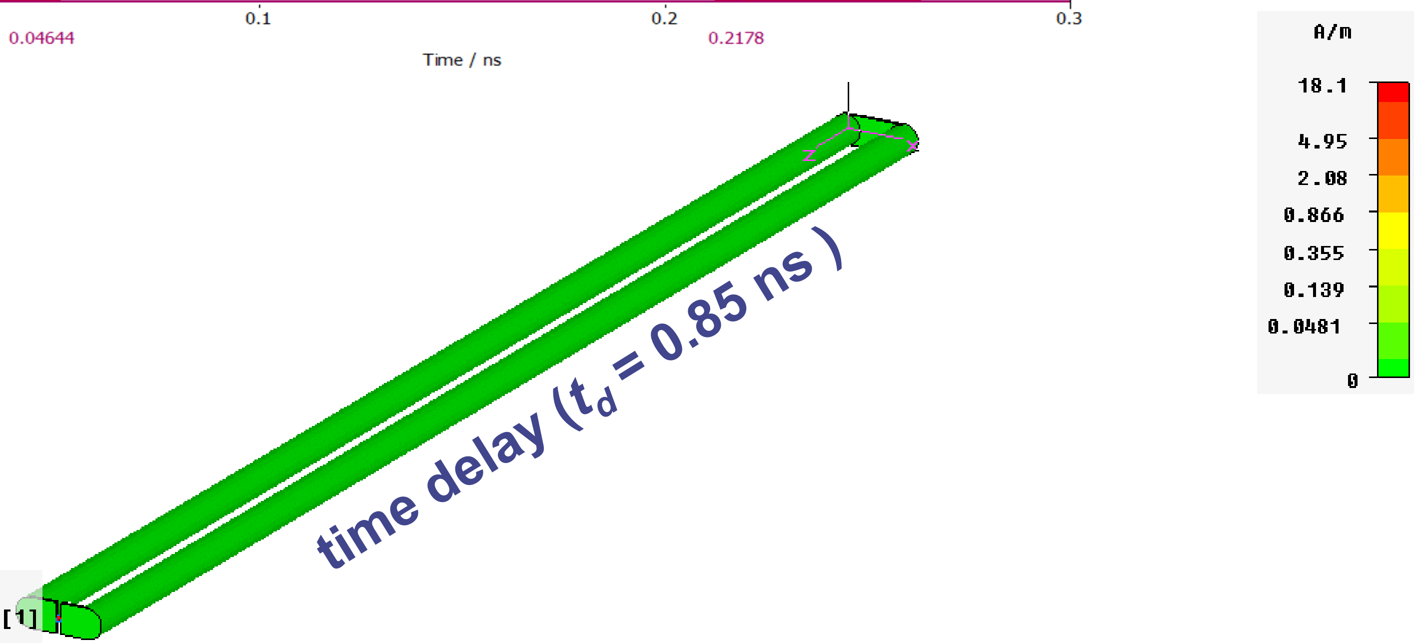
time delay (t_d = 0.85 ns)

Type	Surface Current (peak)
Monitor	h-field (t=0..end(1e-9)) [1]
Component	Abs
Maximum-3d	25.4365 A/m at 0 / 2.5 / 0.5
Sample	1 / 141
Time	0

Transmission Line Behavior

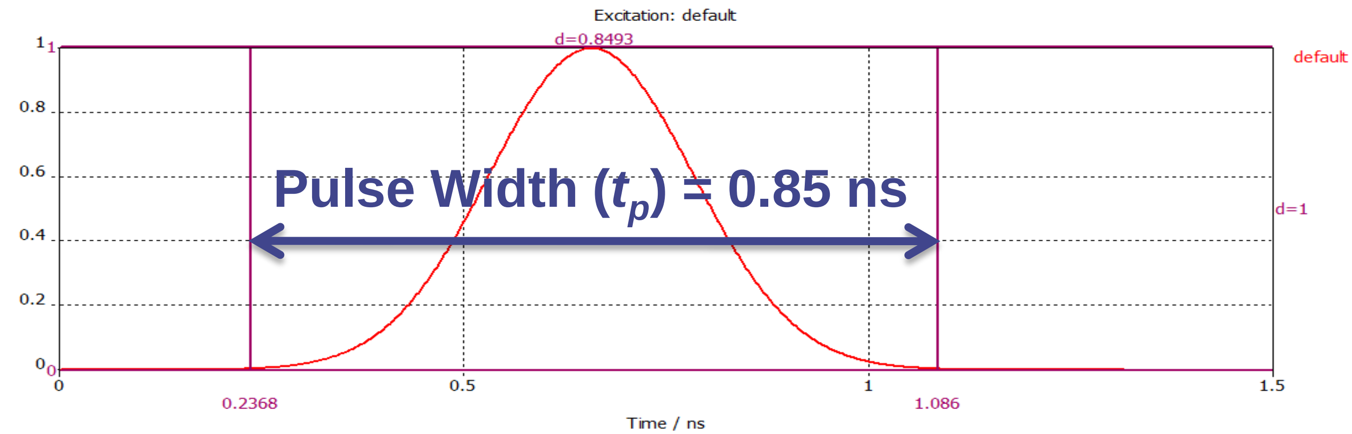


$$@t_p = \frac{1}{5} t_d$$



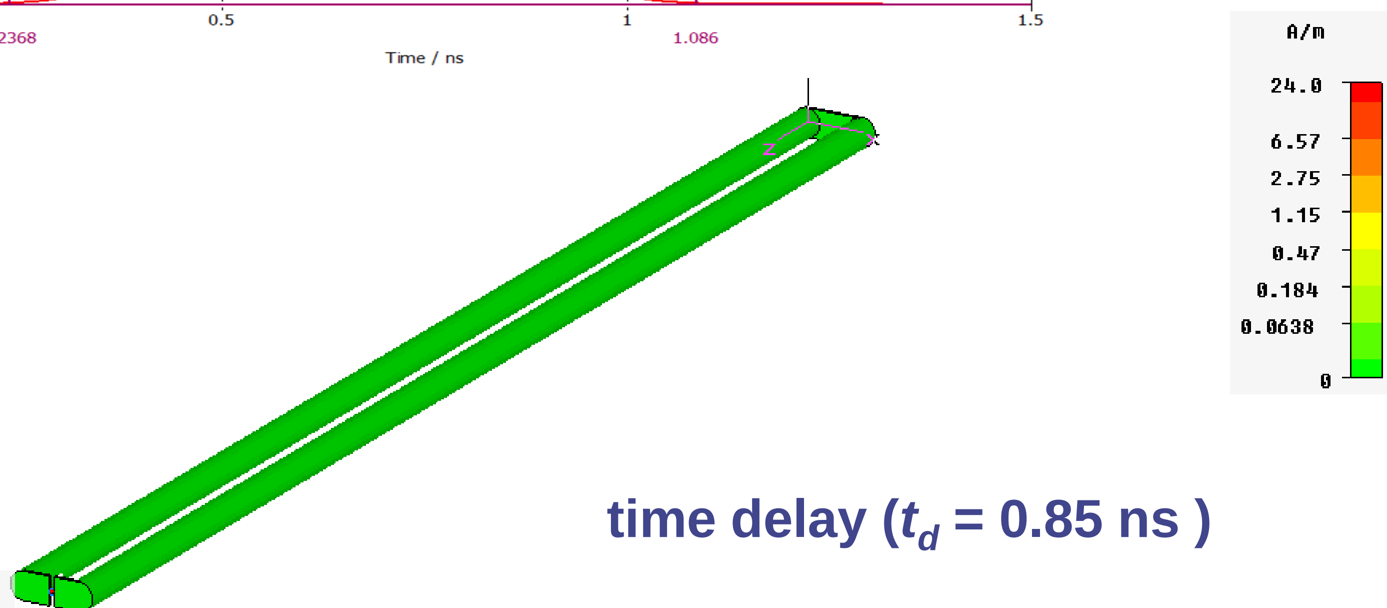
Type	Surface Current (peak)
Monitor	h-field (t=0..end(1e-9)) [1]
Component	Abs
Maximum-3d	18.1429 A/m at 0 / 2.5 / 0.5
Sample	1 / 31
Time	0

Pulse Propagation



CST

$$@t_p = t_d$$



time delay (t_d = 0.85 ns)

Type	Surface Current (peak)
Monitor	h-field (t=0..end(1e-9)) [1]
Component	Abs
Maximum-3d	24.0457 A/m at 0 / 2.5 / 0.5
Sample	1 / 31
Time	0

Guidelines – Distributed *vs.* Lumped Element Behavior

Use transmission line theory <u>if</u>:

- Very conservative: $t_r < 10t_d$ (analogous to **λ/10** in frequency domain)
- Good engineering approximation: $t_r < 6t_d$

  (H. Johnson and M. Graham, *High-Speed Digital Design: A Handbook of Black Magic*, Prentice Hall PTR.)

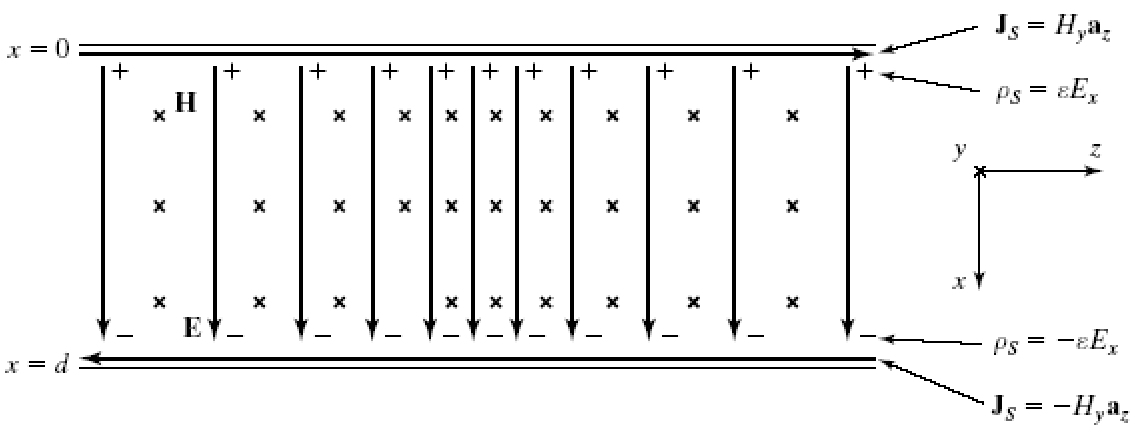
- Approximate calculations: $t_r < 4t_d$ (analogous to **λ/4** in frequency domain)

6.1 Transmission Line
Parallel-plate line

- Let us now consider the uniform plane electromagnetic wave propagating in the z-direction and having an x-component only of the electric field and a y-component only of the magnetic field, that is,

$$\mathbf{E} = E_x(z,t)\mathbf{a}_x$$
$$\mathbf{H} = H_y(z,t)\mathbf{a}_y$$

- place perfectly conducting sheets in two planes at x = 0 and x = d



Why does the wave still propagate?

6.1 Transmission Line
Parallel-plate line

Since the electric field is completely normal and the magnetic field is completely tangential to the sheets, the two boundary conditions the perfect conductors are satisfied, and, hence, the wave will simply propagate.

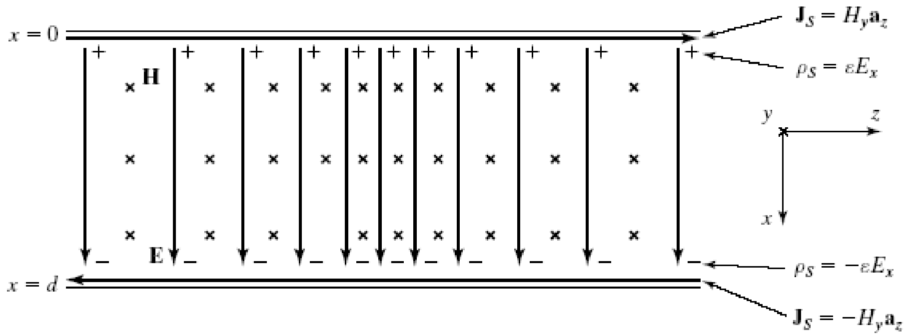
What are the remaining two boundary conditions?

$$[\rho s]_{x=0} = [\mathbf{a}_n \cdot \mathbf{D}]_{x=0} = \mathbf{a}_x \cdot \varepsilon E_x \mathbf{a}_x = \varepsilon E_x$$

$$[\rho s]_{x=d} = [\mathbf{a}_n \cdot \mathbf{D}]_{x=d} = -\mathbf{a}_x \cdot \varepsilon E_x \mathbf{a}_x = -\varepsilon E_x$$

$$[\mathbf{J}s]_{x=0} = [\mathbf{a}_n \times \mathbf{H}]_{x=0} = \mathbf{a}_x \times H_y \mathbf{a}_y = H_y \mathbf{a}_z$$

$$[\mathbf{J}s]_{x=d} = [\mathbf{a}_n \times \mathbf{H}]_{x=d} = -\mathbf{a}_x \times H_y \mathbf{a}_y = -H_y \mathbf{a}_z$$

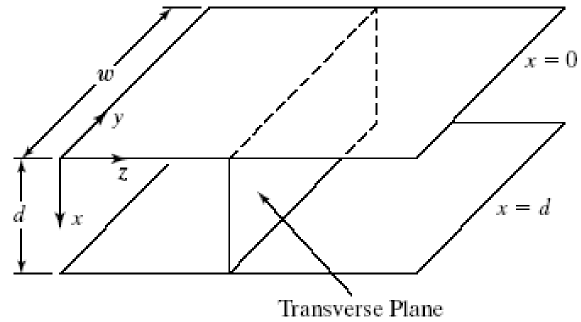


The wave propagation along the transmission line is supported by charges and currents on the plates, varying with time and distance along the line.

6.1 Transmission Line
finite size parallel-plate line

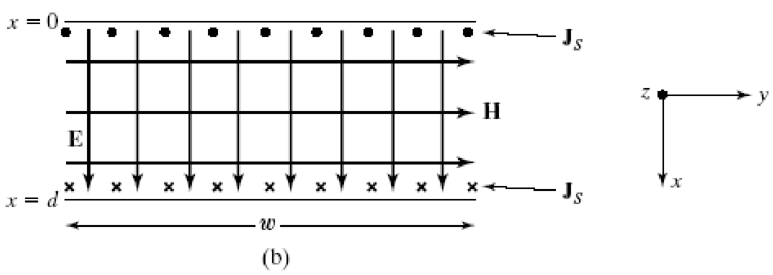
Neglect fringing of the fields at the edges or assume that the structure is part of a much larger-sized configuration.

What important parameters can we define?



Voltage

$$V(z,t) = \int_{x=0}^{d} E_x(z,t)dx = E_x(z,t)\int_{x=0}^{d} dx = dE_x(z,t)$$



Current

$$I(z,t) = \int_{y=0}^{w} Js(z,t)dy = \int_{y=0}^{w} H_y(z,t)dy = H_y(z,t)\int_{y=0}^{w} dy$$
$$= wH_y(z,t)$$

What about the power?

6.1 Transmission Line
finite size parallel-plate line

$$
\begin{aligned}
P(z,t) &= \int_{\substack{transverse\\plane}} (\mathbf{E}\times\mathbf{H})\cdot d\mathbf{S} \\
&= \int_{x=0}^{d}\int_{y=0}^{w} E_x(z,t)H_y(z,t)\mathbf{a}_z \cdot dxdy\mathbf{a}_z \\
&= \int_{x=0}^{d}\int_{y=0}^{w} \frac{V(z,t)}{d}\frac{I(z,t)}{w}dxdy \\
&= V(z,t)I(z,t)
\end{aligned}
$$

Familiar relationship employed in circuit theory.

6.1 Transmission Line
transmission-line equations

Ex and Ey satisfy the two differential equations (see sec4.4)

$$\frac{\partial E_x}{\partial z} = -\frac{\partial B_y}{\partial t} = -\mu \frac{\partial H_y}{\partial t}$$

$$\frac{\partial H_y}{\partial z} = -\sigma E_x - \varepsilon \frac{\partial E_x}{\partial t} = -\varepsilon \frac{\partial E_x}{\partial t}$$

$$E_x = \frac{V}{d}$$

$$H_y = \frac{I}{w}$$

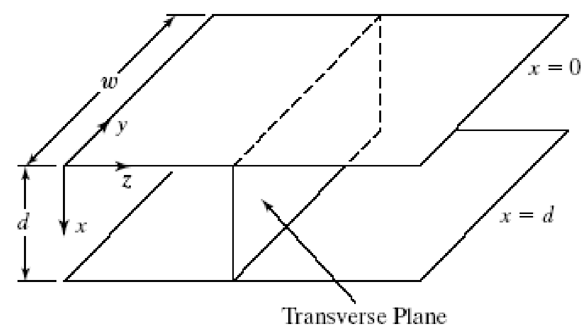
They characterize the wave propagation along the line in terms of line voltage and line current instead of in terms of the fields.

6.1 Transmission Line
transmission-line equations

Definitions of inductance and capacitance ?

L: The ratio of the magnetic flux per unit length at any value of *z* to the line current at that value of *z*.

$$L = \frac{\mu H_y d}{H_y w} = \frac{\mu d}{w}$$



C: The ratio of the magnitude of the charge per unit length on either plate at any value of *z* to the line voltage at that value of *z*.

$$C = \frac{\varepsilon E_x w}{E_x d} = \frac{\varepsilon w}{d}$$

Are L and C purely dependent on the dimensions of the line?
Are they dependent on the Ex and Hy fields?
Are L and C mutually independent?

6.1 Transmission Line
transmission-line equations

$$LC = \mu\varepsilon$$

Replacing now the quantities in parentheses in (6.8a) and (6.8b) by L and C, respectively

$$\frac{\partial V}{\partial z} = -L\frac{\partial I}{\partial t}$$

$$\frac{\partial I}{\partial z} = -C\frac{\partial V}{\partial t}$$

These equations permit us to discuss wave propagation along the line in terms of circuit quantities instead of in terms of field quantities.

It should, however, not be forgotten that the actual phenomenon is one of electromagnetic waves guided by the conductors of the line.

6.1 Transmission Line

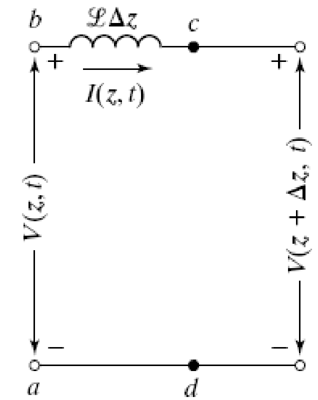
Distributed Equivalent Circuits

Transmission line equations -> transmission line equivalent circuits

$$\lim_{\Delta z \to 0} \frac{V(z+\Delta z,t)-V(z,t)}{\Delta z} = -\mathcal{L} \frac{\partial I(z,t)}{\partial t}$$

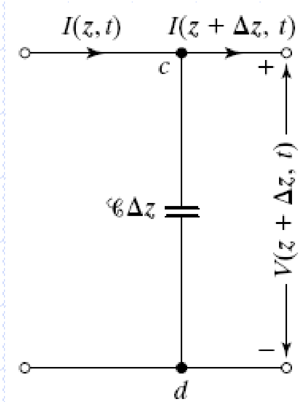
$$V(z+\Delta z,t)-V(z,t) = -\mathcal{L}\,\Delta z \frac{\partial I(z,t)}{\partial t}$$

This equation can be represented by the circuit equivalent shown in Fig. 6.3(a), since it satisfies Kirchhoff's voltage law written around the loop *abcda*.



$$\lim_{\Delta z \to 0} \frac{I(z+\Delta z,t)-I(z,t)}{\Delta z} = \lim_{\Delta z \to 0}\left[-\mathcal{C} \frac{\partial V(z+\Delta z,t)}{\partial t}\right]$$

$$I(z+\Delta z,t)-I(z,t) = -\mathcal{C}\,\Delta z \frac{\partial V(z+\Delta z,t)}{\partial t}$$

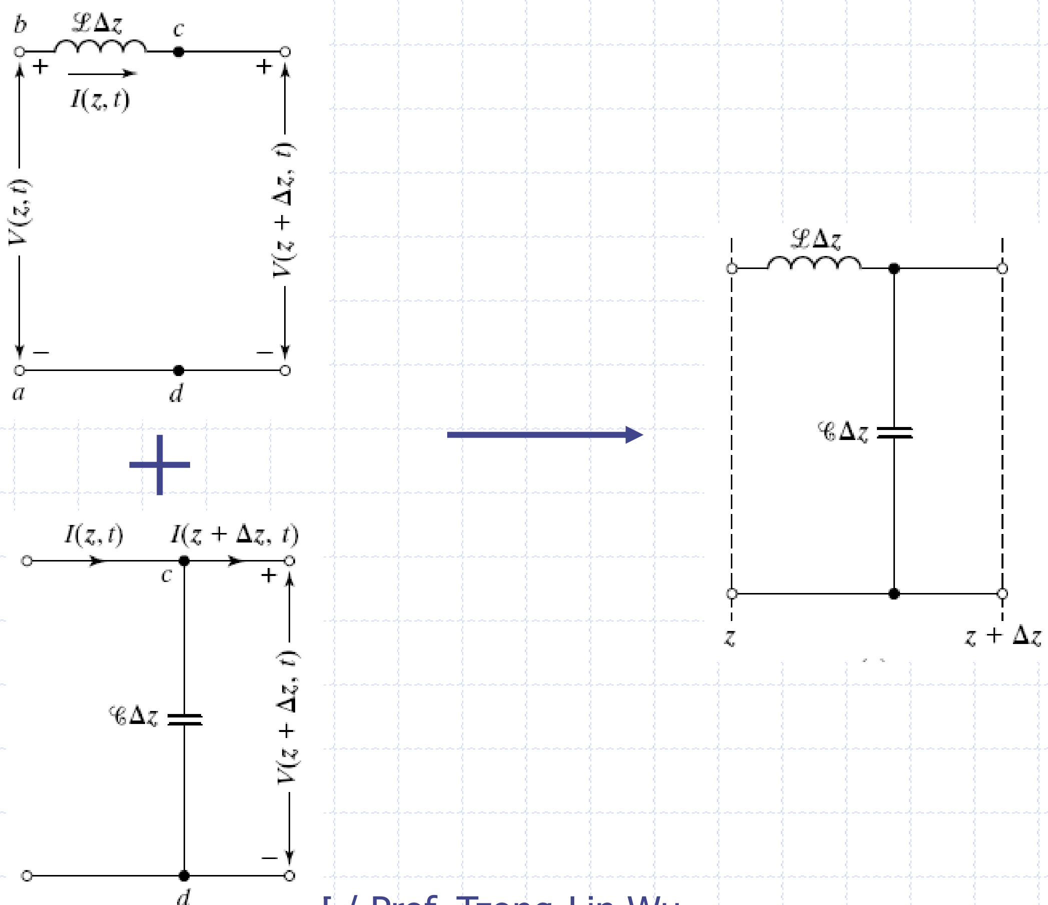


This equation can be represented by the circuit equivalent shown in Fig. 6.3(b), since it satisfies Kirchhoff's current law written for node *c*.

6.1 Transmission Line

Distributed Equivalent Circuits

Combining the two equations, we then obtain the equivalent circuit shown in Fig. 6.3(c) for a section Δz of the line.



6.1 Transmission Line

Distributed Equivalent Circuits

It then follows that the circuit representation for a portion of length *l* of the line consists of an infinite number of such sections in cascade, as shown in Fig. 6.4.

Such a circuit is known as a *distributed circuit* as opposed to the *lumped circuits* that are familiar in circuit theory.

The distributed circuit notion arises from the fact that the inductance and capacitance are distributed uniformly and overlappingly along the line.

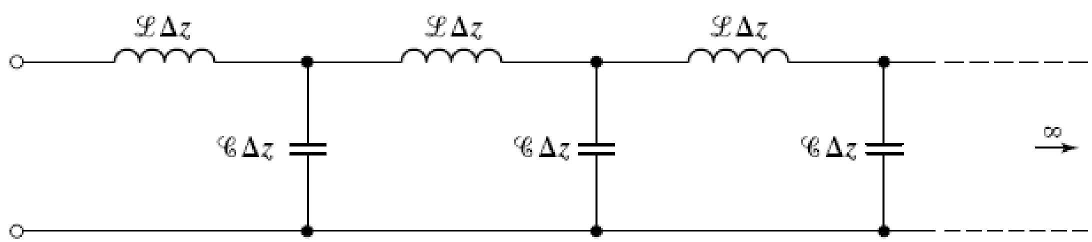


FIGURE 6.4

Distributed circuit representation of a transmission line.

6.1 Transmission Line

Distributed Equivalent Circuits

Energy consideration -> distributed-circuit concept

If we consider a section of the line, the energy stored in the electric field in this section is given by

$$W_e = \frac{1}{2}\varepsilon E_x^2(\text{volume}) = \frac{1}{2}\varepsilon E_x^2(dw\Delta z)$$
$$= \frac{1}{2}\frac{\varepsilon w}{d}(E_x d)^2\Delta z = \frac{1}{2}C\ \Delta z V^2$$

The energy stored in the magnetic field in that section is given by

$$W_m = \frac{1}{2}\mu H_y^2(\text{volume}) = \frac{1}{2}\mu H_y^2(dw\Delta z)$$
$$= \frac{1}{2}\frac{\mu d}{w}(H_y w)^2\Delta z = \frac{1}{2}L\ \Delta z I^2$$

We note that L and C are elements associated with energy storage in the magnetic field and energy storage in the electric field, respectively, for a given infinitesimal section of the line.

Since these phenomena occur continuously and since they overlap, the inductance and capacitance must be distributed uniformly and overlappingly along the line.

6.1 Transmission Line

TEM waves

TEM wave = uniform plane wave ??

In the general case of a line having conductors with arbitrary cross sections, the fields consist of both x- and y-components and are dependent on x- and y-coordinates in addition to the z coordinate.

$$\mathbf{E} = E_x(x, y, z, t)\mathbf{a}_x + E_y(x, y, z, t)\mathbf{a}_y$$
$$\mathbf{H} = H_x(x, y, z, t)\mathbf{a}_x + H_y(x, y, z, t)\mathbf{a}_y$$

These fields are no longer uniform in x and y but are directed entirely transverse to the direction of propagation, that is, the z-axis, which is the axis of the transmission line. Hence, they are known as *transverse electromagnetic waves*, or *TEM waves*.

The uniform plane waves are simply a special case of the transverse electromagnetic waves.

6.1 Transmission Line

TEM waves

Note: The transmission-line equations (6.12a) and (6.12b) and the distributed equivalent circuit of Fig. 6.4 hold for all transmission lines made of perfect conductors and perfect dielectric, that is, for all lossless transmission lines.

What are their differences for different kind of transmission lines?

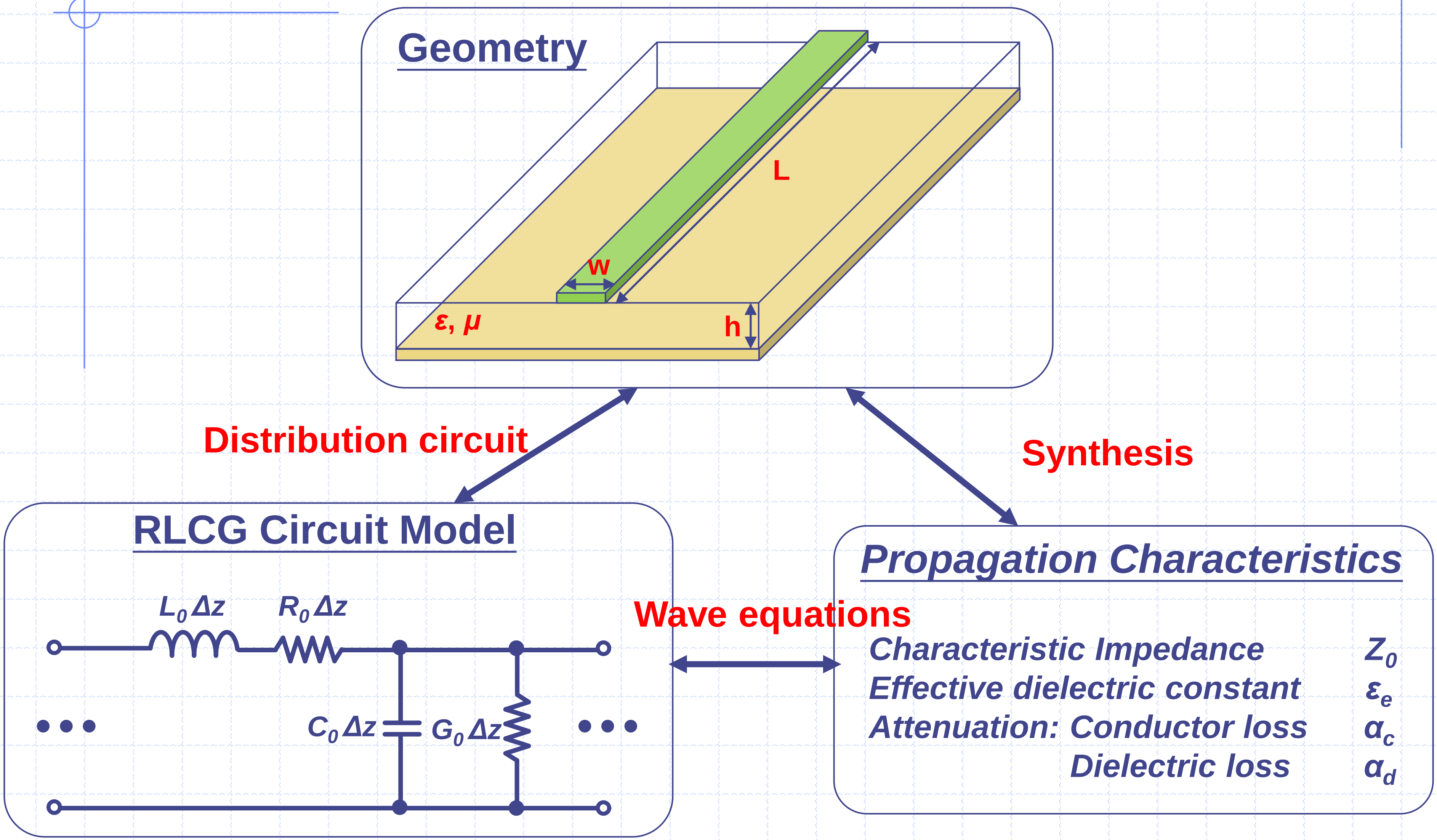
The only differences are the line parameters L and C, which are dependent on the geometry of the lines.

The values of L and C are the same as those obtainable from static field considerations. Why?

Since there is no *z*-component of **H**, the electric-field distribution in any given transverse plane at any given instant of time is the same as the static electric-field distribution resulting from the application of a potential difference between the conductors equal to the line voltage in that plane at that instant of time.

Similarly, since there is no *z*-component of **E**, the magnetic-field distribution in any given transverse plane at any given instant of time is the same as the static magnetic-field distribution resulting from current flow on the conductors equal to the line current in that plane at that instant of time.

Transmission Line Triangle



6.1 Transmission Line

General Solutions

The transmission line equations 6.12a and 6.12b

The general solutions are

$$V(z,t) = Af(t - z\sqrt{LC}) + Bg(t + z\sqrt{LC})$$ Why?

$$I(z,t) = \frac{1}{\sqrt{L/C}}[Af(t - z\sqrt{LC}) - Bg(t + z\sqrt{LC})]$$

These solutions represent voltage and current traveling waves propagating along the +z and –z with velocity

$$v_p = \frac{1}{\sqrt{LC}}$$

Why is it the velocity?

6.1 Transmission Line

General Solutions

We, however, know that the velocity of propagation in terms of the dielectric parameters is given by

$$v_p = \frac{1}{\sqrt{\mu\varepsilon}}$$

Therefore, it follows that

$$LC = \mu\varepsilon$$

6.1 Transmission Line

Characteristic impedance

We now define the *characteristic impedance* of the line to be

$$Z_0 = \sqrt{\frac{L}{C}}$$

so that (6.15a) and (6.15b) become

$$V(z,t) = Af(t - \frac{z}{v_p}) + Bg(t + \frac{z}{v_p})$$

$$I(z,t) = \frac{1}{Z_0}[Af(t - \frac{z}{v_p}) - Bg(t + \frac{z}{v_p})]$$

What is the physical meaning of the characteristic impedance?

The characteristic impedance is the ratio of the voltage to current in the (+) wave or the negative of the same ratio for the (-) wave.

Note: It is analogous to the intrinsic impedance of the dielectric medium but not necessarily equal to it.

6.1 Transmission Line

Characteristic Impedance

An example for parallel-plate line

$$Z_o = \sqrt{\frac{\mu d / w}{\varepsilon w / d}}$$
$$= \eta \frac{d}{w}$$

- Why is it not equal to η?
- Is the characteristic impedance always real?

Other forms for the characteristic impedance and propagation velocity

$$Z_0 = \frac{\sqrt{\mu\varepsilon}}{C}$$

$$Z_0 = \frac{\sqrt{\varepsilon_r}}{cC}$$

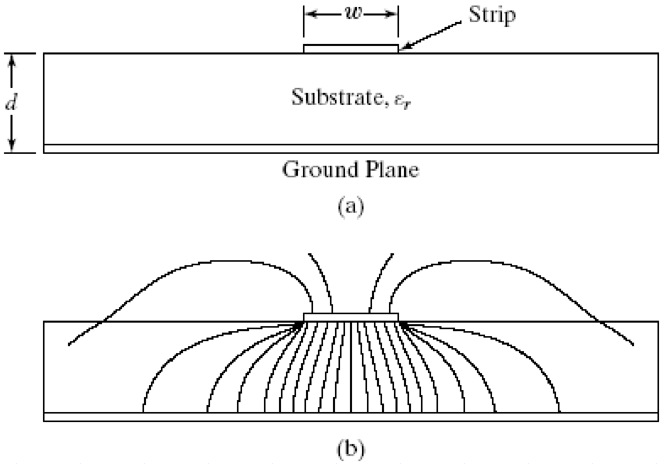
$$v_p = \frac{c}{\sqrt{\varepsilon_r}}$$

, for homogeneous and non-magnetic medium.

6.1 Transmission Line

Microstrip Line

If the cross section of a transmission line involves more than one dielectric, the situation corresponds to **inhomogeneity**.
An example of this type of line is the microstrip line.



Note: it does not support the TEM wave because it is not possible to satisfy the boundary condition of equal phase velocities parallel to the air-dielectric interface with pure TEM waves

How can we solve the propagation characteristics of the inhomogeneous transmission line?

We assume the inhomogeneity has no effect on the per-unit-length inductance L and the propagation is TEM.

Then, we solve two parameters:

6.1 Transmission Line

Microstrip Line

C_0 is the capacitance per unit length of the line with all the dielectrics replaced by free space from static field consideration.

C is the capacitance per unit length of the line with the dielectrics in place and computed from static field considerations.

$$Z_o = \sqrt{\frac{L}{C}} = \sqrt{\frac{L\,C_o}{C\,C_o}} = \frac{1}{c\sqrt{C\,C_o}} \qquad Z_o = \frac{\sqrt{\varepsilon_{eff}}}{cC}$$

$$v_p = \frac{1}{\sqrt{L\,C}} = \frac{1}{\sqrt{L\,C_o}}\sqrt{\frac{C_o}{C}} = c\sqrt{\frac{C_o}{C}} \qquad v_p = \frac{c}{\sqrt{\varepsilon_{eff}}}$$

We define the effective relative permittivity as

$$\varepsilon_{eff} = \frac{C}{C_0}$$

How can we find the C_0 and C?

6.1 Transmission Line

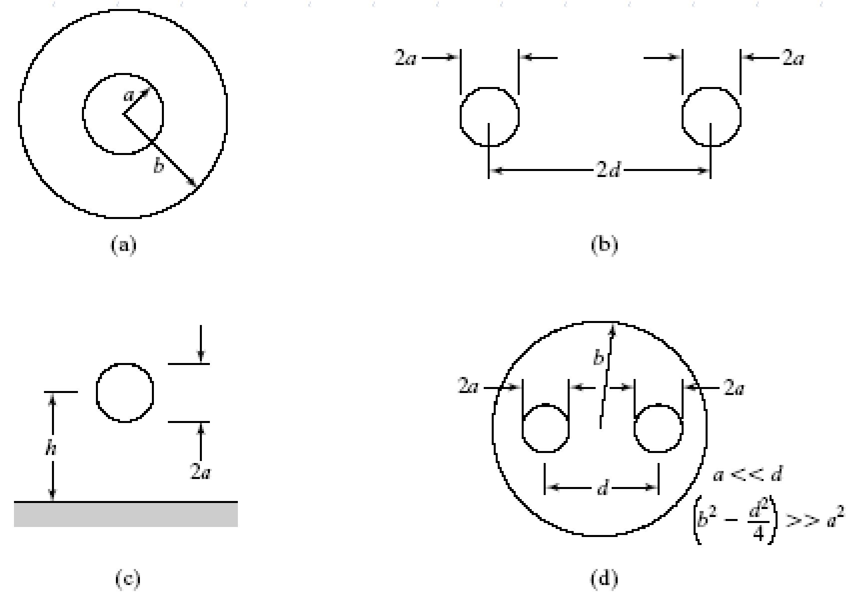
Microstrip Line

A. Analytical Techniques

The analytical techniques are based on the closed-form solution of Laplace's equation, subject to the boundary conditions, or the equivalent of such a solution.
We have already discussed these techniques in Sections 5.3 and 5.4 for several configurations.

Table 6.1 Expressions for Characteristic Impedance for the Lines of Fig. 6.6

Description	Figure	Z_0
Coaxial cable	6.6(a)	$\frac{\eta}{2\pi}\ln\frac{b}{a}$
Parallel-wire line	6.6(b)	$\frac{\eta}{\pi}\cosh^{-1}\frac{d}{a}$
Single wire above ground plane	6.6(c)	$\frac{\eta}{2\pi}\cosh^{-1}\frac{h}{a}$
Shielded parallel-wire line	6.6(d)	$\frac{\eta}{\pi}\ln\frac{d(b^2-d^2/4)}{a(b^2+d^2/4)}$



B. Numerical Techniques

When a closed-form solution is not possible or when the approximation permitting
a closed-form solution breaks down, numerical techniques can be employed. (chapter 11)

6.1 Transmission Line

Microstrip Line

C. Graphical Technique

For a line with arbitrary cross section and involving a homogeneous dielectric, an approximate value of C and, hence, of Z_0 can be determined by constructing a *field map*, that is, a graphical sketch of the direction lines of the electric field and associated equipotential lines between the conductors. (Chapter 11.5)

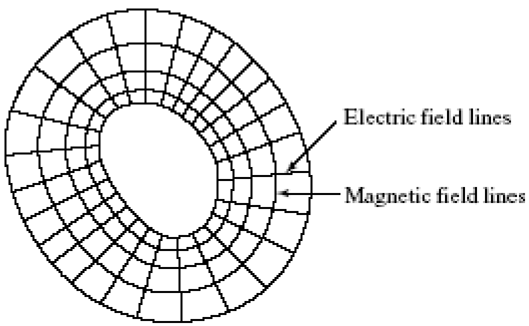


FIGURE 6.7
Example of field map for a line of arbitrary cross section.

6.2 Line terminated by Resistive Loads

Notation

General solutions to the transmission-line equations for the lossless line

$$V(z,t) = V^+(t - \frac{z}{v_p}) + V^-(t + \frac{z}{v_p})$$

$$I(z,t) = \frac{1}{Z_0}[V^+(t - \frac{z}{v_p}) - V^-(t + \frac{z}{v_p})]$$

$$V = V^+ + V^-$$

$$I = \frac{1}{Z_0}(V^+ - V^-)$$

$$I = I^+ + I^-$$

$$I^+ = \frac{V^+}{Z_0}$$

$$I^- = -\frac{V^-}{Z_0}$$

What is the meaning of the negative sign of the current?

6.2 Line terminated by Resistive Loads

Notation

$$P^{+} = V^{+}I^{+} = V^{+}\left(\frac{V^{+}}{Z_0}\right) = \frac{(V^{+})^2}{Z_0}$$

$$P^{-} = V^{-}I^{-} = V^{-}\left(-\frac{V^{-}}{Z_0}\right) = -\frac{(V^{-})^2}{Z_0}$$

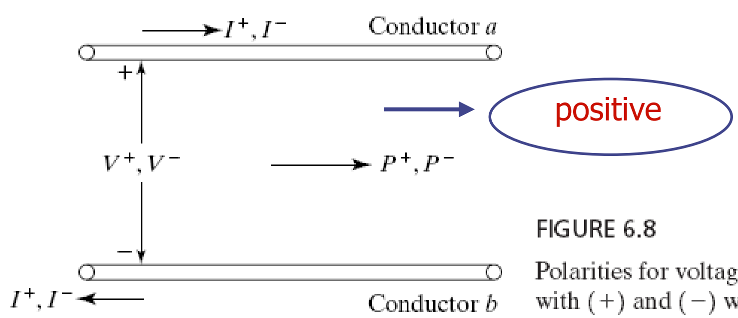


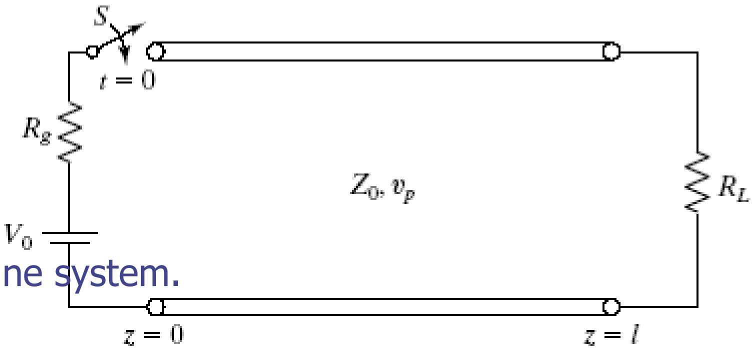
FIGURE 6.8
Polarities for voltages and currents associated with (+) and (−) waves.

The minus sign in (6.32b) indicates that is negative, and, hence, the wave power actually flows in the negative *z*-direction, as it should.

6.2 Line terminated by Resistive Loads

Excitation by a constant source

Let us now consider a line of length l terminated by a load resistance R_l and driven by a constant voltage source in series with internal resistance R_g



We assume that no voltage and current exist on the line for t < 0 and the switch *S* is closed at t = 0.

A step transient wave is thus launched to the transmission line system.

What the impedance does the transient wave see at t = 0?

What is the voltage amplitude on the transmission line at t = 0?

When the switch *S* is closed at a (+) wave originates at z = 0 and travels toward the load. Let the voltage and current of this (+) wave be V^+ and I^-, respectively.

6.2 Line terminated by Resistive Loads

Excitation by a constant source

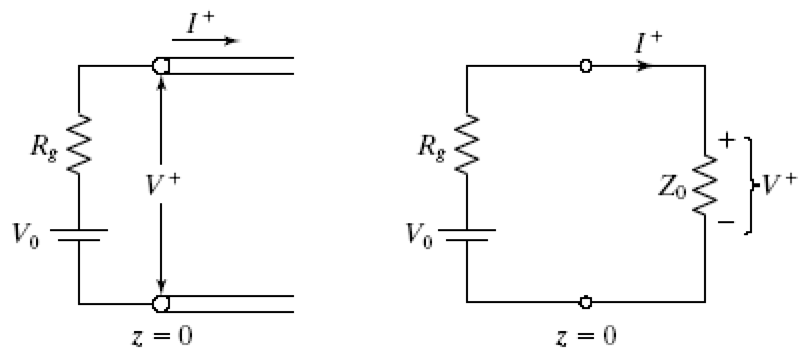
The various quantities must satisfy the boundary condition, that is, Kirchhoff's voltage law around the loop.

$$V_0 - I^+ R_g - V^+ = 0$$

$$V_0 - \frac{V^+}{Z_0} R_g - V^+ = 0$$

$$V^+ = V_0 \frac{Z_0}{R_g + Z_0}$$

$$I^+ = \frac{V^+}{Z_0} = \frac{V_0}{R_g + Z_0}$$



- When does the V+ wave reach the termination R_L?
- What is the boundary conditions at the terminal of load resistance?
- And what happen for the V+ wave?

6.2 Line terminated by Resistive Loads

Reflection coefficient

The wave travels toward the load and reaches the termination at t = l / V_p

The voltage across the load resistance to be equal to the current through it times its value R_L, but the voltage-to-current ratio in the (+) wave is equal to Z_0.

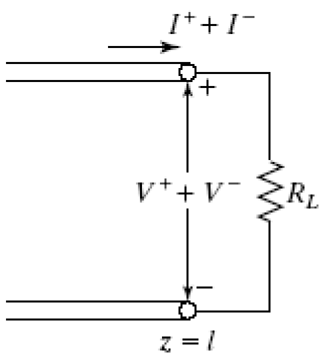
To resolve this inconsistency, there is only one possibility, which is the setting up of a (-) wave, or a reflected wave V-.

To satisfy the boundary condition,

$$V^+ + V^- = R_L(I^+ + I^-)$$

$$V^+ + V^- = R_L\left(\frac{V^+}{Z_0} - \frac{V^-}{Z_0}\right)$$

$$V^- = V^+ \frac{R_L - Z_0}{R_L + Z_0}$$



Voltage reflection coefficient

$$\Gamma = \frac{V^-}{V^+} = \frac{R_L - Z_0}{R_L + Z_0}$$

Current reflection coefficient

$$\frac{I^-}{I^+} = \frac{-V^-/Z_0}{V^+/Z_0} = -\frac{V^-}{V^+} = -\Gamma$$

6.2 Line terminated by Resistive Loads

Reflection coefficient

We observe that this wave V- travels back toward the source and it reaches there at t = 2// $V_{p.}$

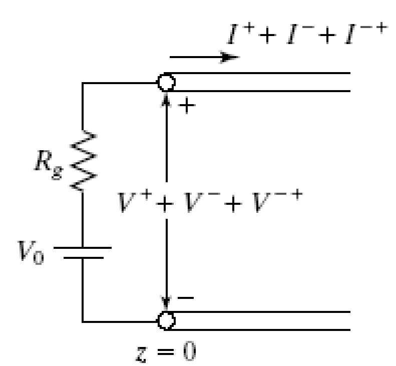
Since the boundary condition at z = 0 which was satisfied by the original wave V+ alone, is then violated, a reflection of the reflection, or a re-reflection, will be set up and it travels toward the load.

To satisfy the boundary condition,

$$V^{+} + V^{-} + V^{-+} = V_0 - R_g (I^{+} + I^{-} + I^{-+})$$

$$V^{+} + V^{-} + V^{-+} = V_0 - \frac{R_g}{Z_0}(V^{+} - V^{-} + V^{-+})$$

$$V^{-+} = V^{-} \frac{R_g - Z_0}{R_g + Z_0}$$



We note that the reflected wave views the source with internal resistance as the internal resistance alone; that is, the voltage source is equivalent to a short circuit insofar as the (-) wave is concerned.

6.2 Line terminated by Resistive Loads

Reflection coefficients for some special cases

- short circuit line, $Z_L = 0$
  - $\Gamma = -1$
- Open circuit line, $Z_L = \infty$
  - $\Gamma = 1$
- Matching circuit line, $Z_L = Z_0$
  - $\Gamma = 0$

6.2 Line terminated by Resistive Loads

Bounce Diagram

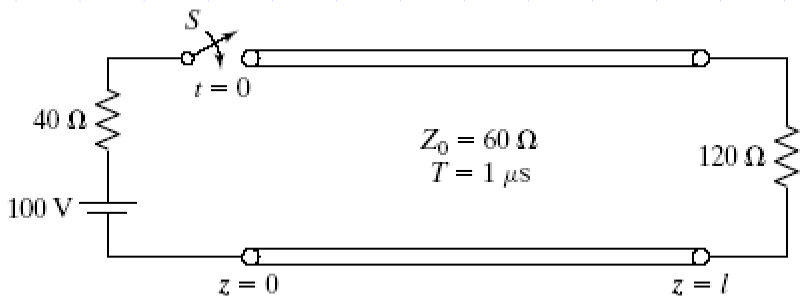
Quantity computation

Voltage carried by the initial (+) wave

Current carried by the initial (+) wave

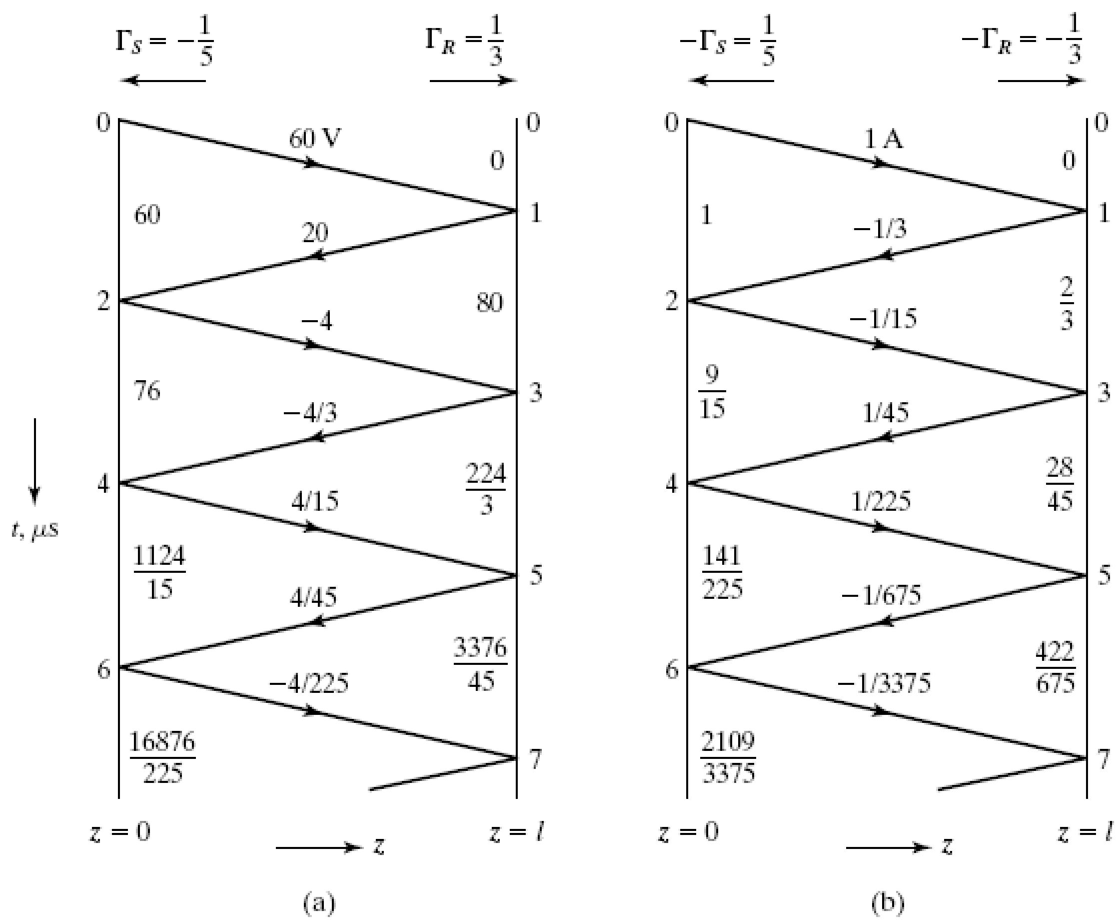
Voltage reflection coefficient at load,

Voltage reflection coefficient at source,



6.2 Line terminated by Resistive Loads

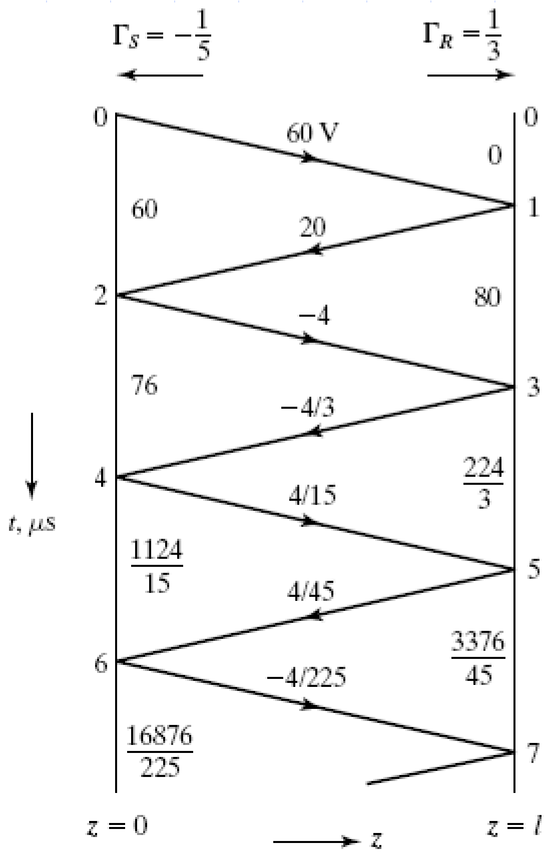
Bounce Diagram

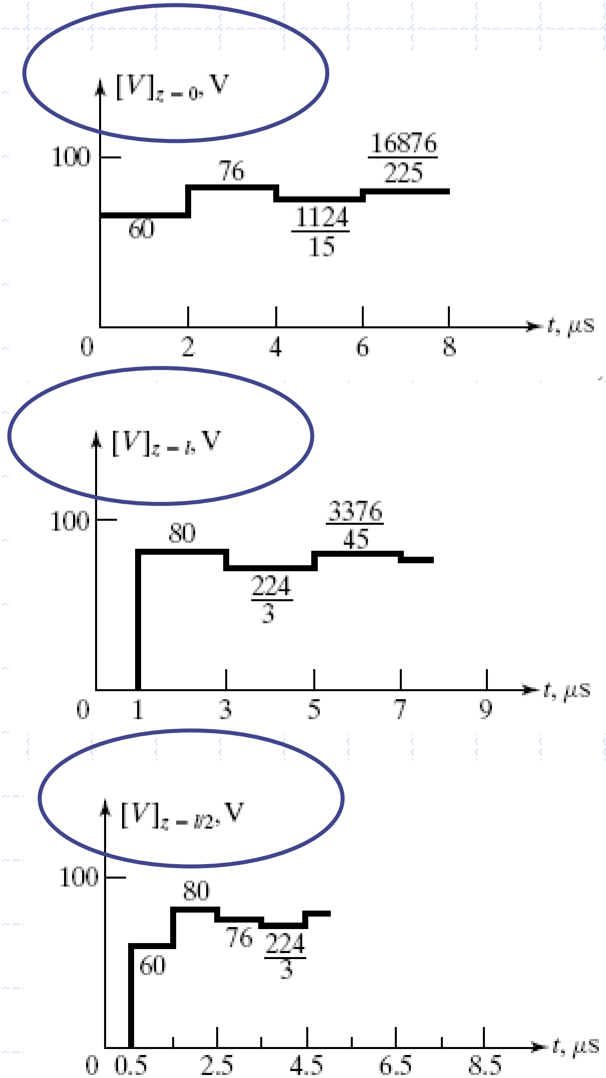


6.2 Line terminated by Resistive Loads

Bounce Diagram

Plots of line voltage versus t (transient)

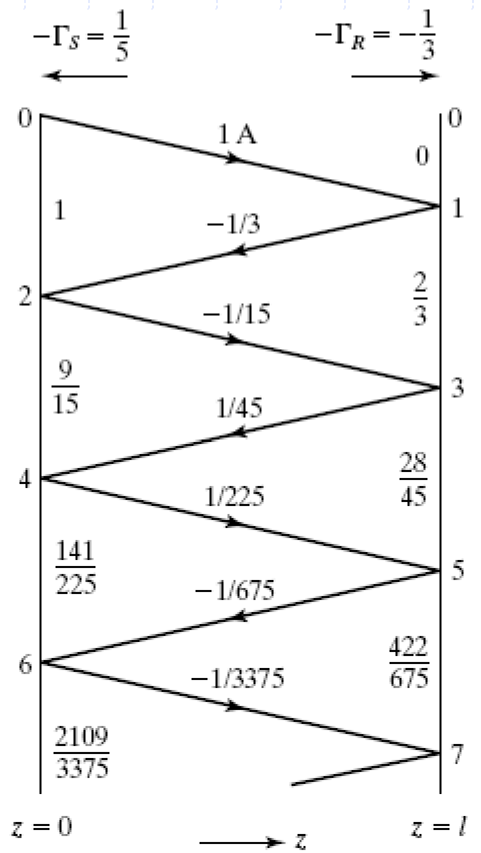


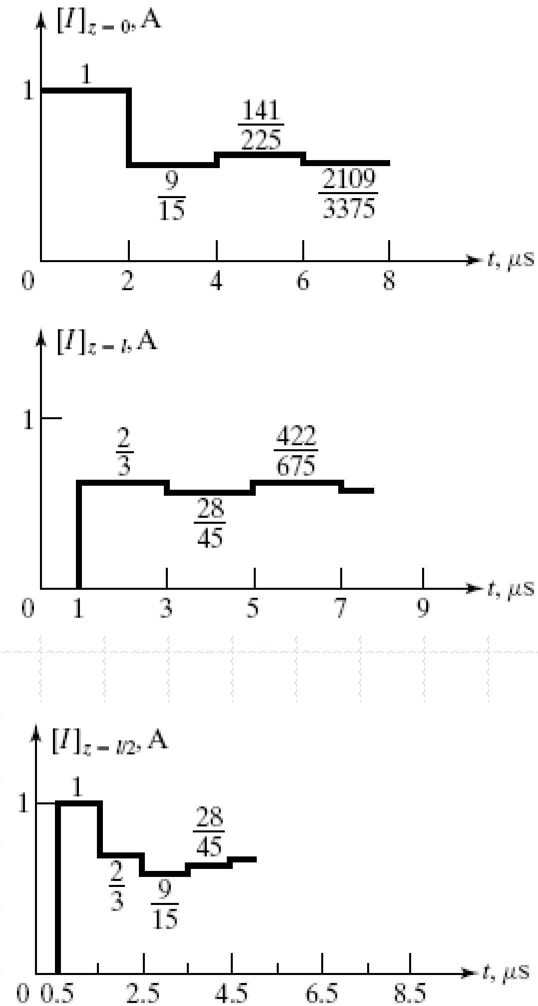


6.2 Line terminated by Resistive Loads

Bounce Diagram

Plots of line current versus t (transient)





The above time-domain are the transient behavior of V and I.
What is the steady state of this switch circuits for V and I?

6.2 Line terminated by Resistive Loads

Bounce Diagram

Steady state

As time progresses, the line voltage and current tend to converge to certain values, which we can expect to be the steady state values.

In the steady state, the situation consists of a single (+) wave, which is actually a superposition of the infinite number of transient (+) waves, and a single (-) wave, which is actually a superposition of the infinite number of transient (-) waves.

$$V_{SS}^{+} = 60 - 4 + \frac{4}{15} - \cdots = 60\left(1 - \frac{1}{15} + \frac{1}{15^2} - \cdots\right) = 56.25\ \text{V}$$

$$I_{SS}^{+} = 1 - \frac{1}{15} + \frac{1}{225} - \cdots = 1 - \frac{1}{15} + \frac{1}{15^2} - \cdots = 0.9375\ \text{A}$$

$$V_{SS}^{-} = 20 - \frac{4}{3} + \frac{4}{45} - \cdots = 20\left(1 - \frac{1}{15} + \frac{1}{15^2} - \cdots\right) = 18.75\ \text{V}$$

$$I_{SS}^{-} = -\frac{1}{3} + \frac{1}{45} - \frac{1}{675} + \cdots = -\frac{1}{3}\left(1 - \frac{1}{15} + \frac{1}{15^2} - \cdots\right) = -0.3125\ \text{A}$$

The steady-state line voltage and current can now be obtained to be

$$V_{SS} = V_{SS}^{+} + V_{SS}^{-} = 75\ \text{V}$$

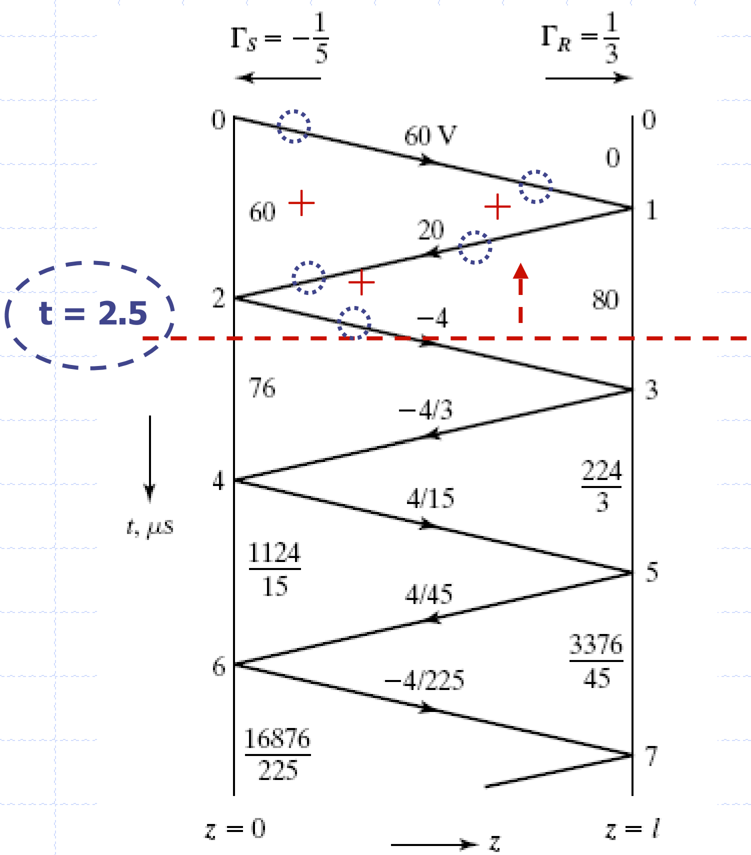
$$I_{SS} = I_{SS}^{+} + I_{SS}^{-} = 0.625\ \text{A}$$

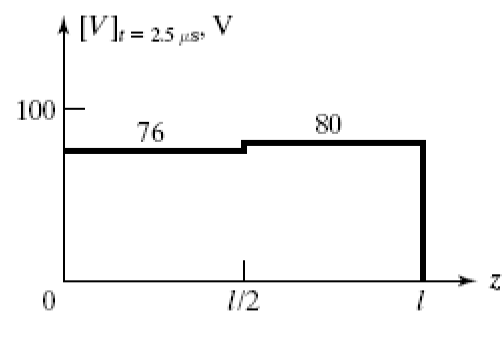
Is there any intuitive concept to explain the results?

6.2 Line terminated by Resistive Loads

Bounce Diagram

Plots of line voltage and current versus z





(a)

6.2 Line terminated by Resistive Loads

Excitation by Pulse Voltage Source

This can be done by representing the pulse as the superposition of two step functions, and superimposing the bounce diagrams for the two sources one on another

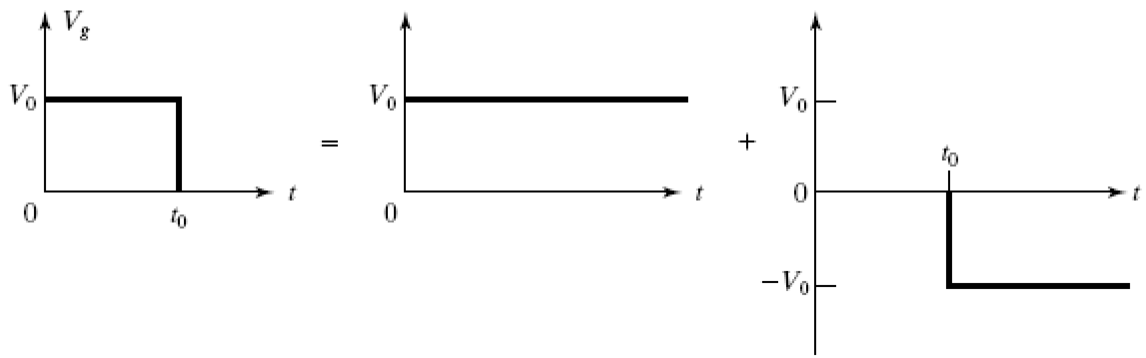


FIGURE 6.17

Representation of a rectangular pulse as the superposition of two step functions.

6.2 Line terminated by Resistive Loads

Excitation by Pulse Voltage Source

Let us assume that the voltage source in the system of Fig. 6.12 is a 100-V rectangular pulse extending from t = 0 to t = 1us and extend the bounce-diagram technique.

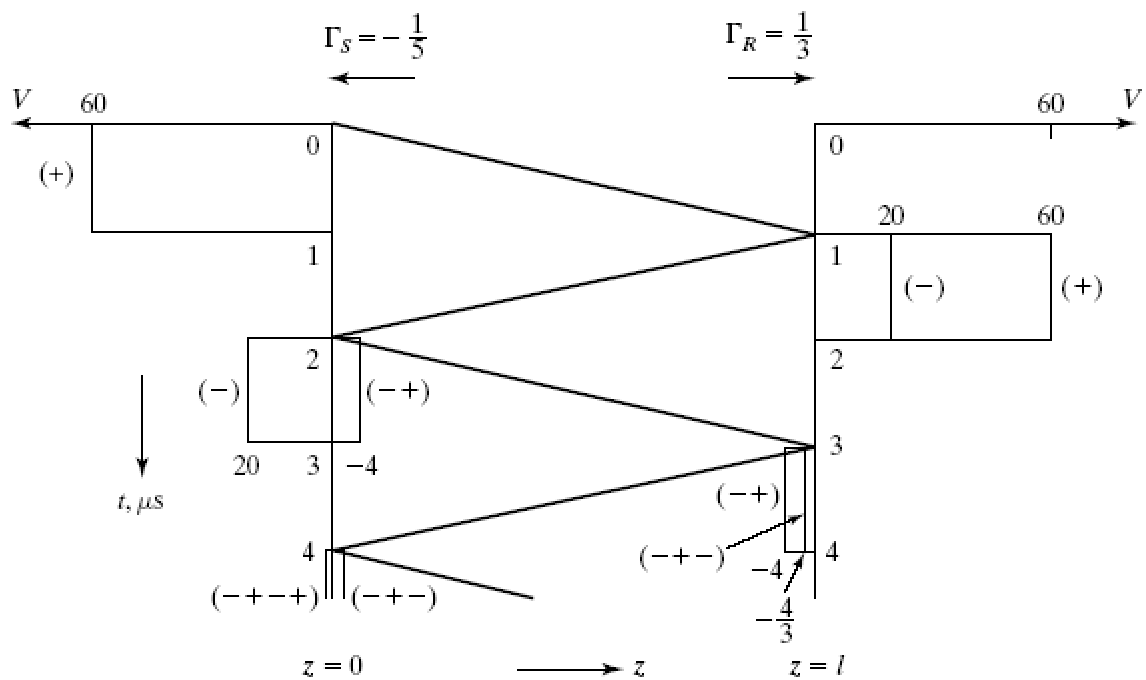


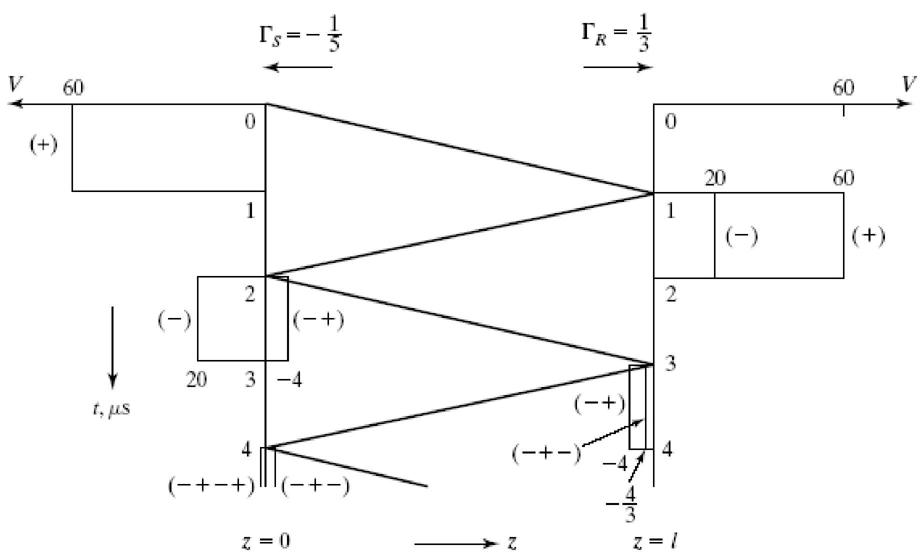
FIGURE 6.18

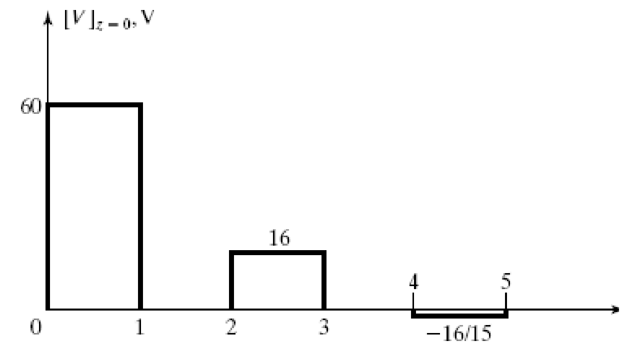
Voltage bounce diagram for the system of Fig. 6.12 except that the voltage source is a rectangular pulse of 1-μs duration from $t = 0$ to $t = 1$ μs.

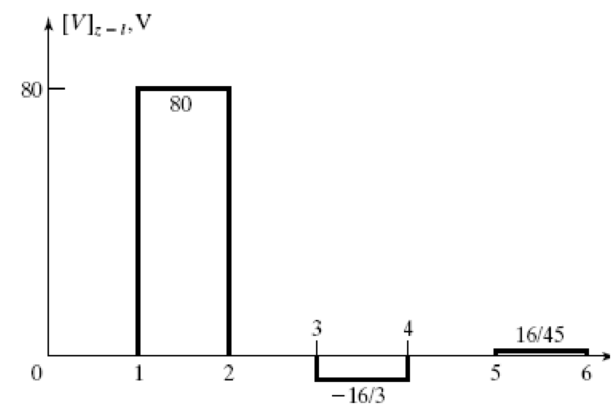
6.2 Line terminated by Resistive Loads

Excitation by Pulse Voltage Source

Line voltage versus time at z = 0 and *z* = *l*



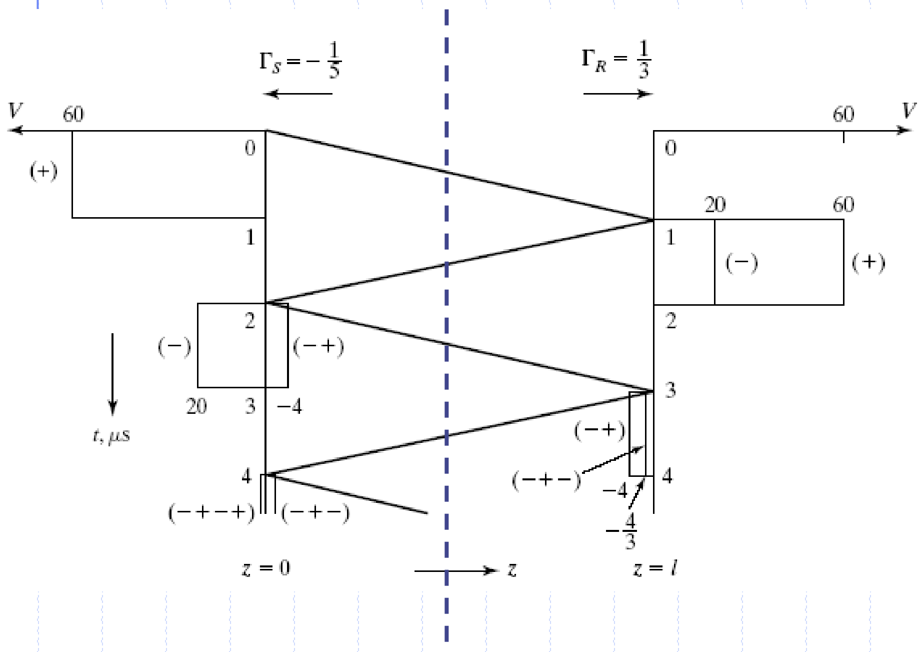


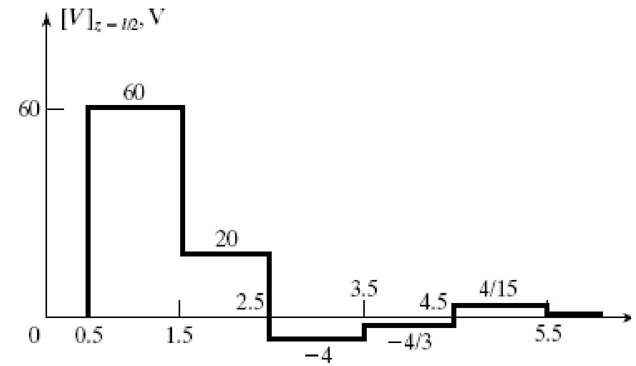


6.2 Line terminated by Resistive Loads

Excitation by Pulse Voltage Source

<u>Line voltage versus time at *z = l/2*</u>



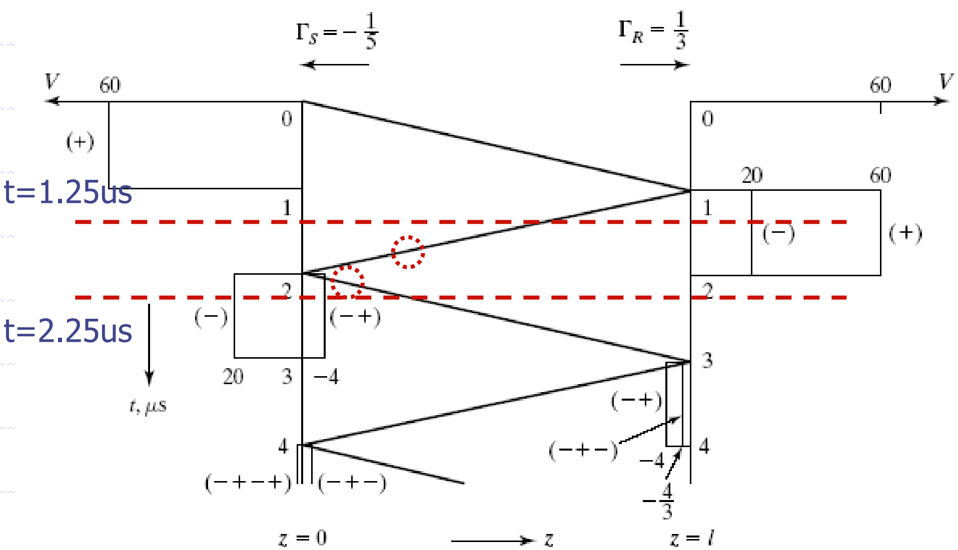


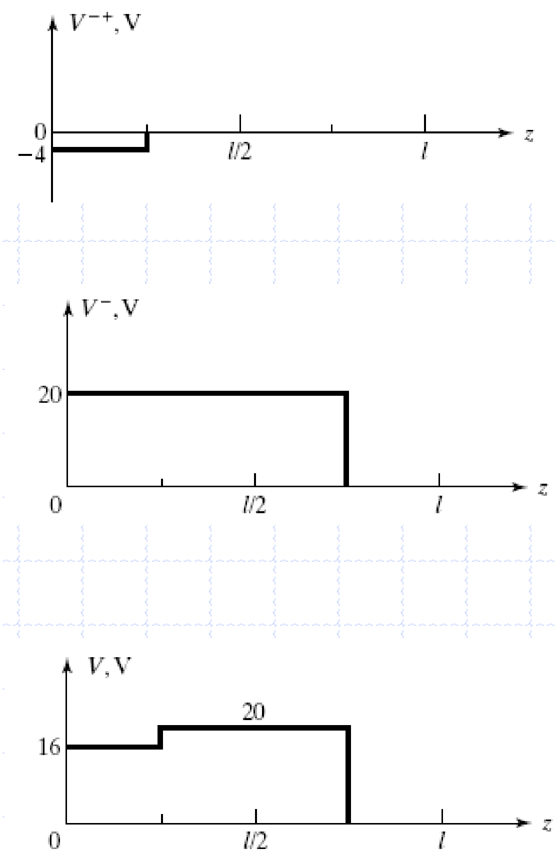
To draw the sketch for *z = l/2,*
we displace the time plots of the (+) waves at z=0 and of the (-) waves at *z = l* forward in time by 0.5us that is, <u>delay them by 0.5us</u> and add them to obtain the plot.

6.2 Line terminated by Resistive Loads

Excitation by Pulse Voltage Source

Sketches of line voltage versus distance (*z*) along the line for fixed values of time





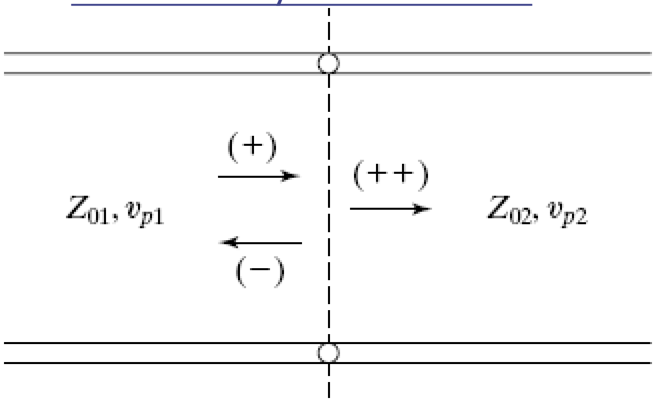
Since the one-way travel time on the line is 1us,
we take the portion from t=2.25 back to t=1.25us.

6.3 Transmission-Line Discontinuity

Junction between two lines

As a (+) wave of voltage *V*+ and current *I*+ is incident on the junction from line 1, what happened?

A reflected wave and a transmitted wave are set up such that the boundary conditions are satisfied.



$$V^{+} + V^{-} = V^{++}$$

$$I^{+} + I^{-} = I^{++}$$

$$\frac{V^{+}}{Z_{01}} - \frac{V^{-}}{Z_{02}} = \frac{V^{++}}{Z_{02}}$$

$$\Gamma = \frac{V^{-}}{V^{+}} = \frac{Z_{02} - Z_{01}}{Z_{02} + Z_{01}}$$

What it the physical meaning?

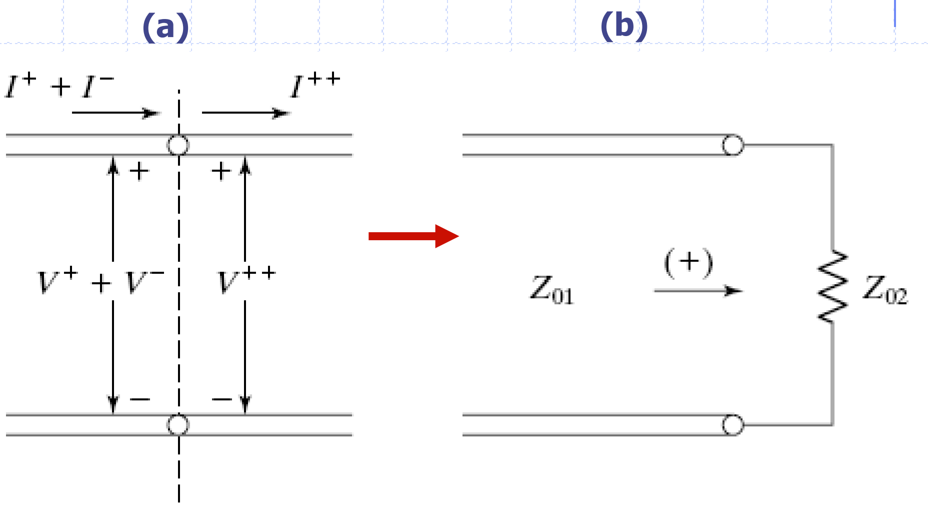
6.3 Transmission-Line Discontinuity

Junction between two lines

Thus, to the incident wave, the transmission line to the right in (a) looks like its characteristic impedance as shown in (b).

Note:
What is the difference between (a) and (b)?

A resistive load (50 ohm for example)
and the transmission line load (Z0 = 50 ohm)



power is dissipated in the load v.s.
the power is transmitted into the line

6.3 Transmission-Line Discontinuity

Transmission Coefficient

The voltage transmission coefficient

$$\tau_V = \frac{V^{++}}{V^{+}} = \frac{V^{+}+V^{-}}{V^{+}} = 1+\frac{V^{-}}{V^{+}} = 1+\Gamma$$

The current transmission coefficient

$$\tau_C = \frac{I^{++}}{I^{+}} = \frac{I^{+}+I^{-}}{I^{+}} = 1+\frac{I^{-}}{I^{+}} = 1-\Gamma$$

Q&A:
The transmitted voltage can be greater than the incident voltage if Γ is positive. Is it possible in real world?

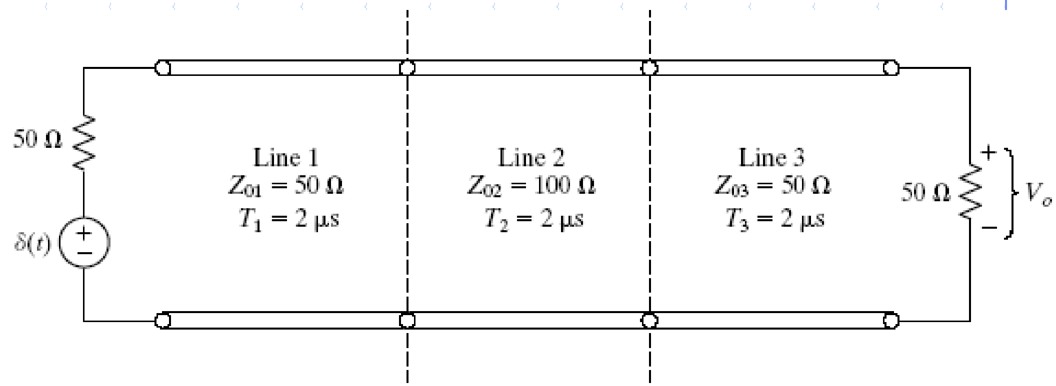
The transmitted power

$$P^{++} = V^{++}I^{++} = V^{+}(1+\Gamma)I^{+}(1-\Gamma)$$
$$= V^{+}I^{+}(1-\Gamma^{2}) = (1-\Gamma^{2})P^{+}$$

What does it imply?

6.3 Transmission-Line Discontinuity

Ex6.3: Unit impulse response and frequency response for a system of three lines in cascade

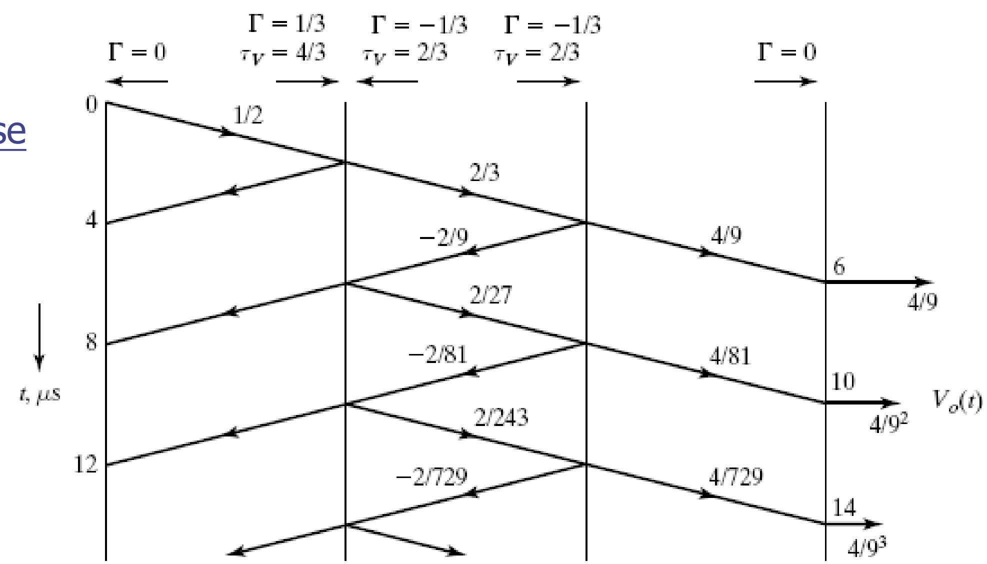


Let us consider the system of three lines in cascade, driven by a unit impulse voltage source

$$V_o(t) = \frac{4}{9}\delta(t-6\times10^{-6}) + \frac{4}{9^2}\delta(t-10\times10^{-6}) + \frac{4}{9^3}\delta(t-14\times10^{-6}) + \cdots$$

$$= \frac{4}{9}\sum_{n=0}^{\infty}\left(\frac{1}{9}\right)^n \delta(t-4n\times10^{-6}-6\times10^{-6})$$

Note that 4/9 is the strength of the first impulse and 1/9 is the multiplication factor for each succeeding impulse.



(b)

In general form:

$$V_0(t) = \frac{4}{9}\sum_{n=0}^{\infty}\left(\frac{1}{9}\right)^n \delta\left[t-2nT_2-(T_1+T_2+T_3)\right]$$

$$= \frac{4}{9}\sum_{n=0}^{\infty}\left(\frac{1}{9}\right)^n \delta\left(t-2nT_2-T_0\right)$$

where $T_1+T_2+T_3 = T_0$

6.3 Transmission-Line Discontinuity

Junction between two lines

Frequency response

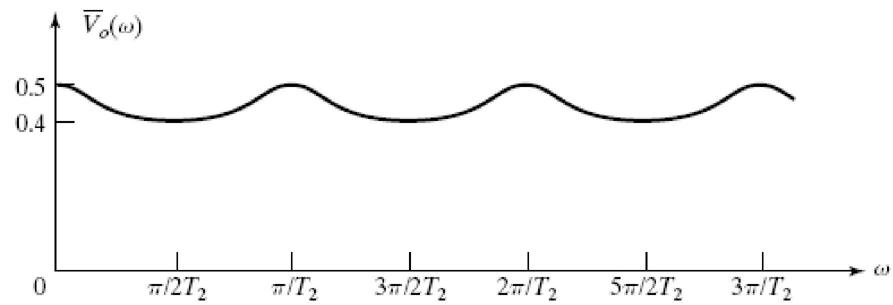
Assume input is sinusoidal

$$V_0(t) = \frac{4}{9}\sum_{n=0}^{\infty}\left(\frac{1}{9}\right)^n \cos\omega\left(t - 2nT_2 - T_0\right)$$

$$\overline{V_0}(\omega) = \frac{4}{9}\sum_{n=0}^{\infty}\left(\frac{1}{9}\right)^n e^{-j\omega(2nT_2+T_0)}$$

$$= \frac{4}{9}e^{-j\omega T_0}\sum_{n=0}^{\infty}\left(\frac{1}{9}e^{-j\omega T_2}\right)^n$$

$$= \frac{(4/9)e^{-j\omega T_0}}{1-(1/9)e^{-j2\omega T_2}}$$



Maximum occurs at $\omega = m\pi / T_2$ its value $\frac{4}{9}/(1-\frac{1}{9}) = 0.5$

Minimum occurs at $\omega = (2m+1)\pi / 2T_2$ its value $\frac{4}{9}/(1+\frac{1}{9}) = 0.4$

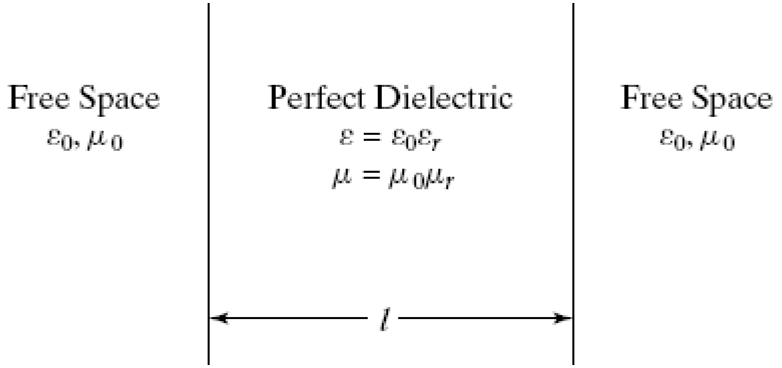
6.3 Transmission-Line Discontinuity

Junction between two lines

Radome : an enclosure for protecting an antenna from the weather while allowing transparency for electromagnetic waves.

A simple, idealized, planar version of the radome is a dielectric slab with free space on either side of it.

The arrangement is equivalent to three lines in cascade, with the characteristic impedances equal to the intrinsic impedances of the media and the velocities of propagation equal to those in the media.



The amplitude versus frequency response is of the same form as previous example, where T_2 is the one-way travel time in the dielectric slab and the maximum is 1 instead of 0.5.
(the factor of 0.5 in previous example is due to voltage drop across the internal resistance of the source in the transmission-line system)

The lowest frequency for which the dielectric slab is completely transparent is $\omega = \pi / T_2 = \pi c / l\sqrt{\mu_r \varepsilon_r}$

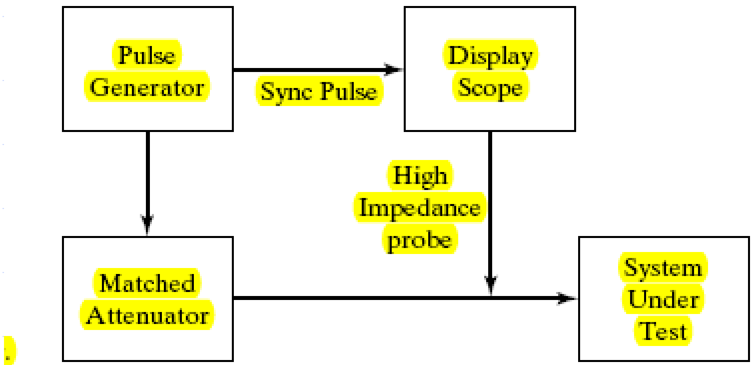
6.3 Transmission-Line Discontinuity

Time-domain reflectometry (TDR)

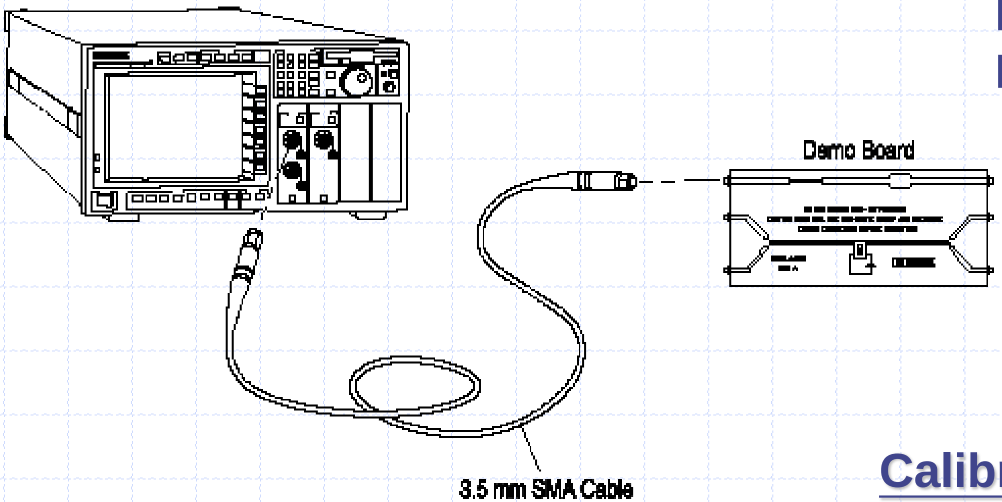
TDR, a technique by means of which discontinuities in transmission-line systems can be located by making measurements with pulses.

What is the purpose of the matched attenuator?

The matched attenuator serves the purpose of absorbing the pulses arriving back from the system so that reflections of those pulses are not produced

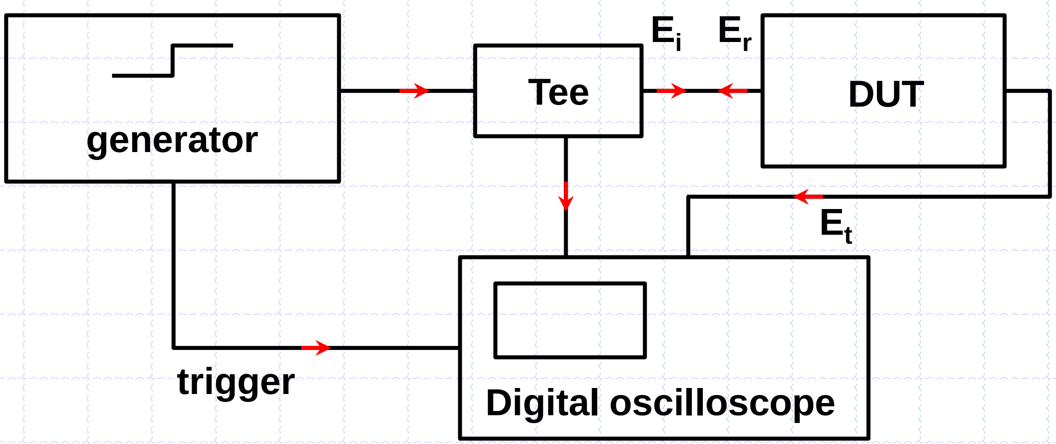


Time Domain Reflectometry



Bandwidth: 20GHz
Min. rising time: 38ps

Calibration of error in TDR measurement is quite important

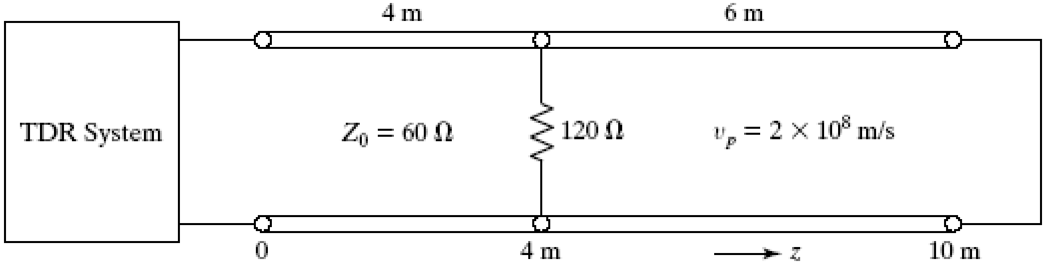


6.3 Transmission-Line Discontinuity

Time-domain reflectometry (TDR)

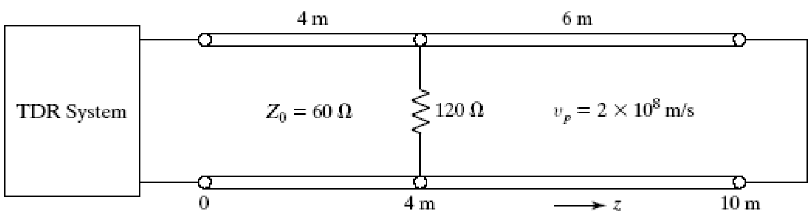
A discontinuity exists at z = 4m and the line is short-circuited at the far end.

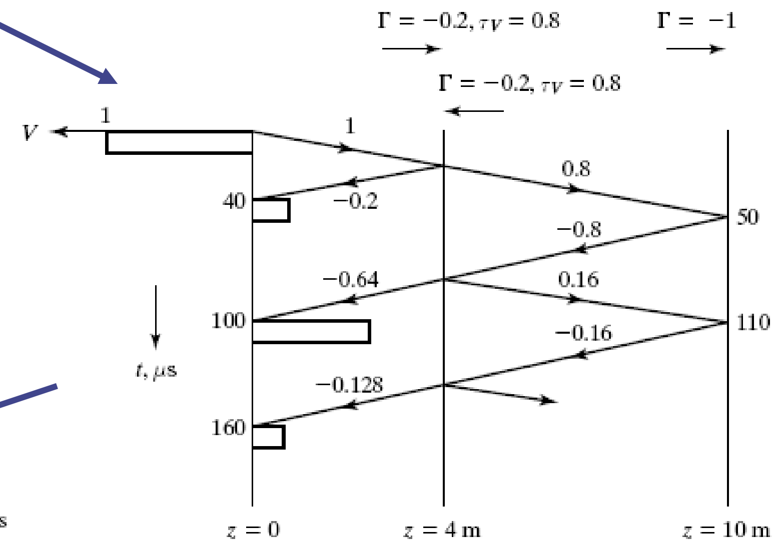
Assuming the TDR pulses to be of amplitude 1 V, duration 10 ns, and repetition rate 10^5Hz.
We shall then discuss how one can deduce the information about the discontinuity from the TDR measurement.

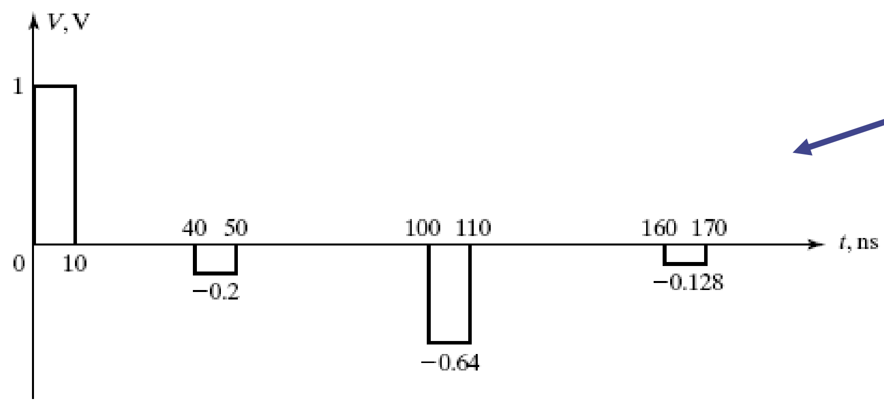


6.3 Transmission-Line Discontinuity

Time-domain reflectometry (TDR)







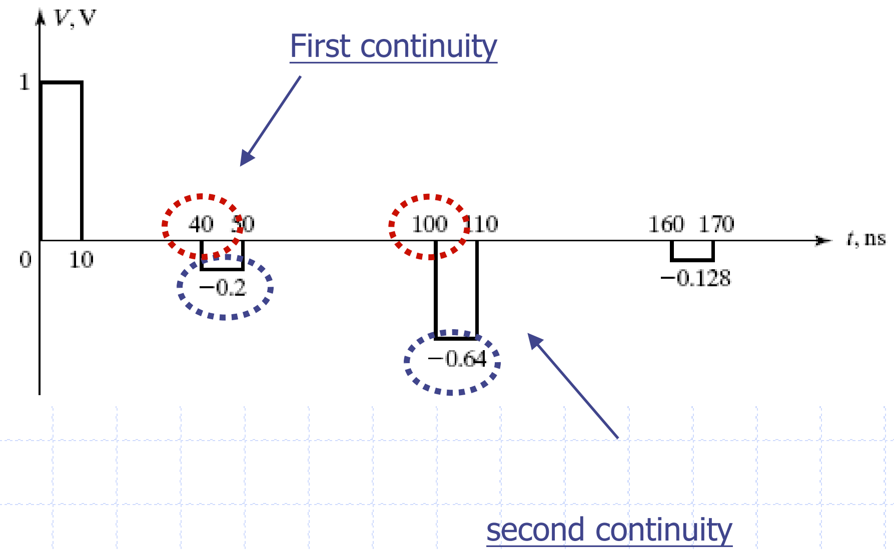
Can you predict the discontinuity situations from the measured voltage waveform on the TDR?

6.3 Transmission-Line Discontinuity

Time-domain reflectometry (TDR)

First discontinuity: how far and how large?

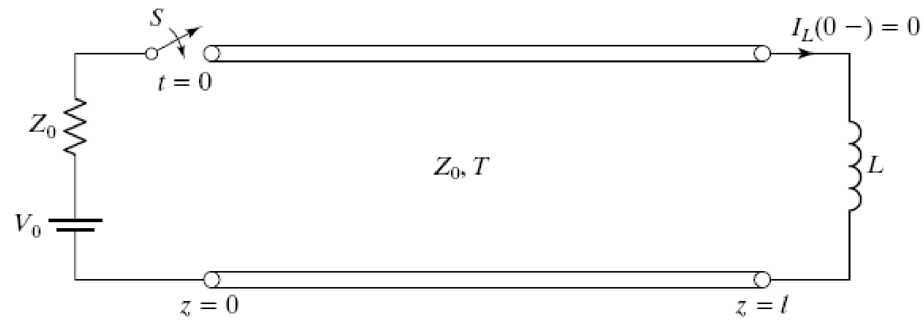
Second discontinuity: how far and how large?



6.4 Line with reactive terminations and discontinuities

Inductive termination

A line of length *l* is terminated by an inductor *L* with zero initial current and a constant-voltage source with internal resistance equal to the characteristic impedance of the line is connected to the line at t = 0.



The boundary condition at $z = l$

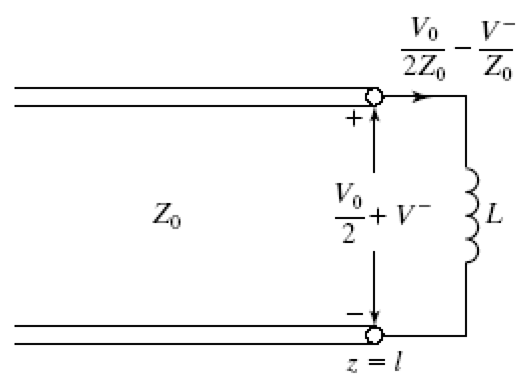
$$\frac{V_0}{2} + V^- = L\frac{d}{dt}\left(\frac{V_0}{2Z_0} - \frac{V^-}{Z_0}\right)$$

$$\frac{L}{Z_0}\frac{dV^-}{dt} + V^- = -\frac{V_0}{2}$$

Initial condition

$$\left[\frac{V_0}{2Z_0} - \frac{V^-}{Z_0}\right]_{t=T} = 0$$

$$\left[V^-\right]_{t=T} = \frac{V_0}{2}$$



Solution:

$$V^- = -\frac{V_0}{2} + Ae^{-(Z_0/L)t}$$

$$A = V_0 e^{(Z_0/L)T}$$

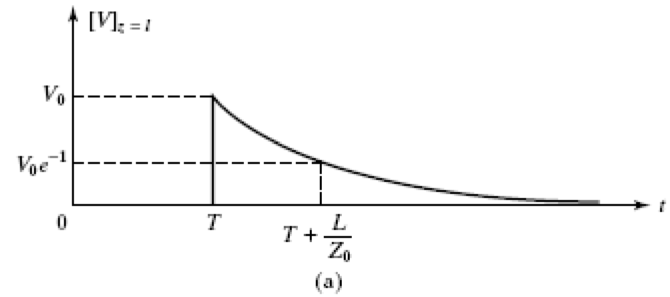
$$V^-(l,t) = -\frac{V_0}{2} + V_0 e^{-(Z_0/L)(t-T)} \quad \text{for } t > T$$

6.4 Line with reactive terminations and discontinuities

Inductive termination

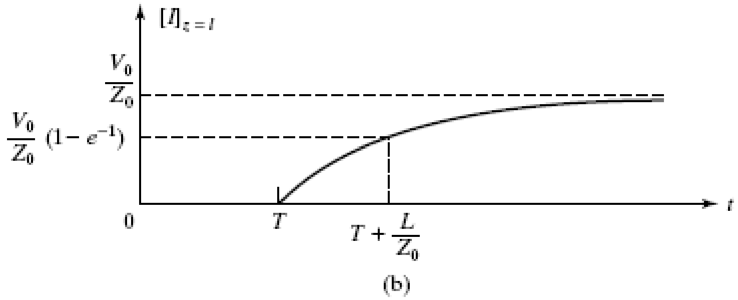
$$V(l,t) = \frac{V_0}{2} + V^-(l,t)$$

$$= \begin{cases} 0 & for \quad t < T \\ V_0 e^{-(Z_0/L)(t-T)} & for \quad t > T \end{cases}$$



$$I(l,t) = \frac{V_0}{2Z_0} + I^-(l,t)$$

$$= \begin{cases} 0 & for \quad t < T \\ (V_0/Z_0)\left[1 - e^{-(Z_0/L)(t-T)}\right] & for \quad t > T \end{cases}$$



Can you predict the waveform behavior by an intuitive physical concept?

–

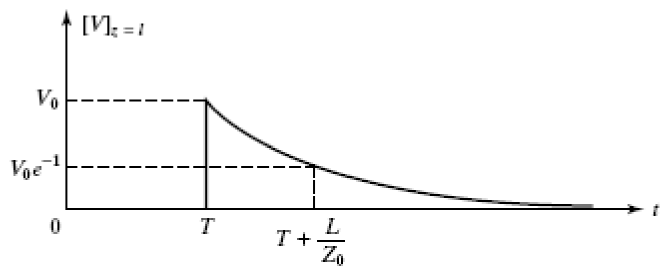
6.4 Line with reactive terminations and discontinuities

Inductive termination

The inductor is looked as

1. Short circuit at steady state t = infinite.
2. Open circuit at initial transient for t = T.

Between t = T and t = infinite, the behavior follows the time constant L/Z_0.



(a)

Reflection at Discontinuity (reactive discontinuity)

- Concept

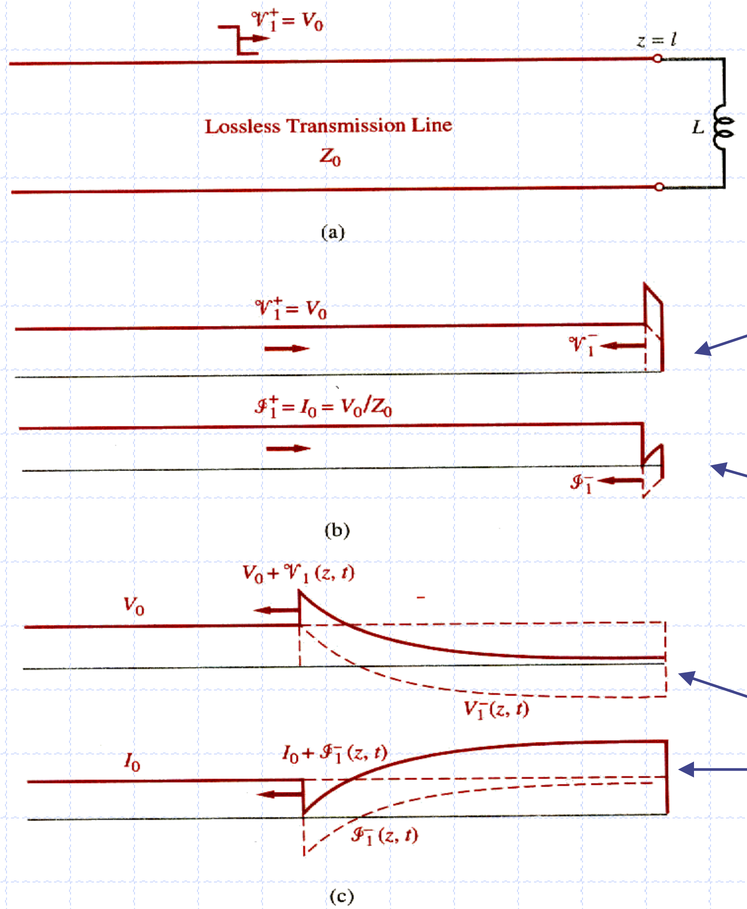


FIGURE 2.24. Reflection from a purely inductive load. The source is assumed to be matched (i.e., no reflections back from the source) and to supply a constant voltage V_0. The distributions of the voltage and current are shown at two different times: (b) immediately after reflection when the inductor behaves like an open circuit, and (c) later in time when the inductor behaves as a short circuit.

- Transient step response of an inductor

Reflection coefficient is depend on frequency

When steady state (DC), inductance acts like a short.

6.4 Line with reactive terminations and discontinuities

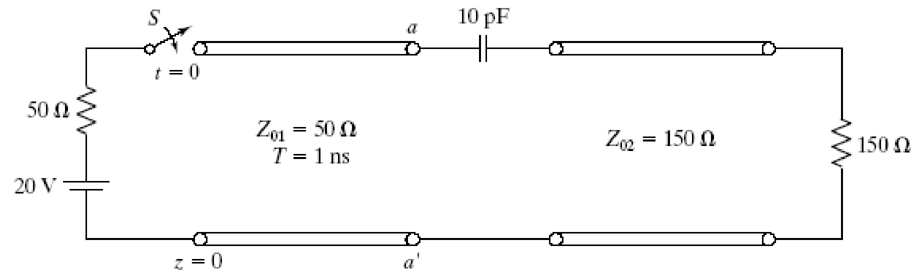
Capacitive discontinuity

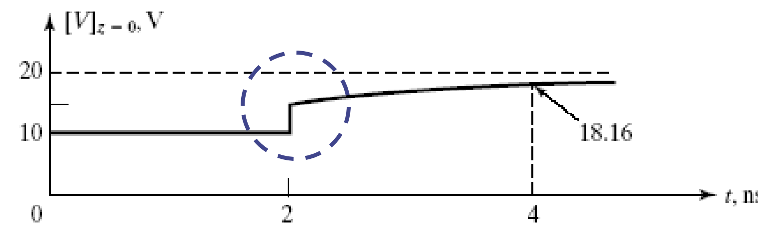
Can you explain the waveform behavior?

Why 15V at t = 2ns?

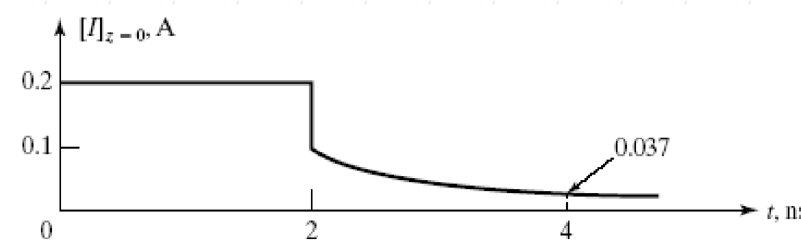
What is the steady state voltage?

What is the time constant?

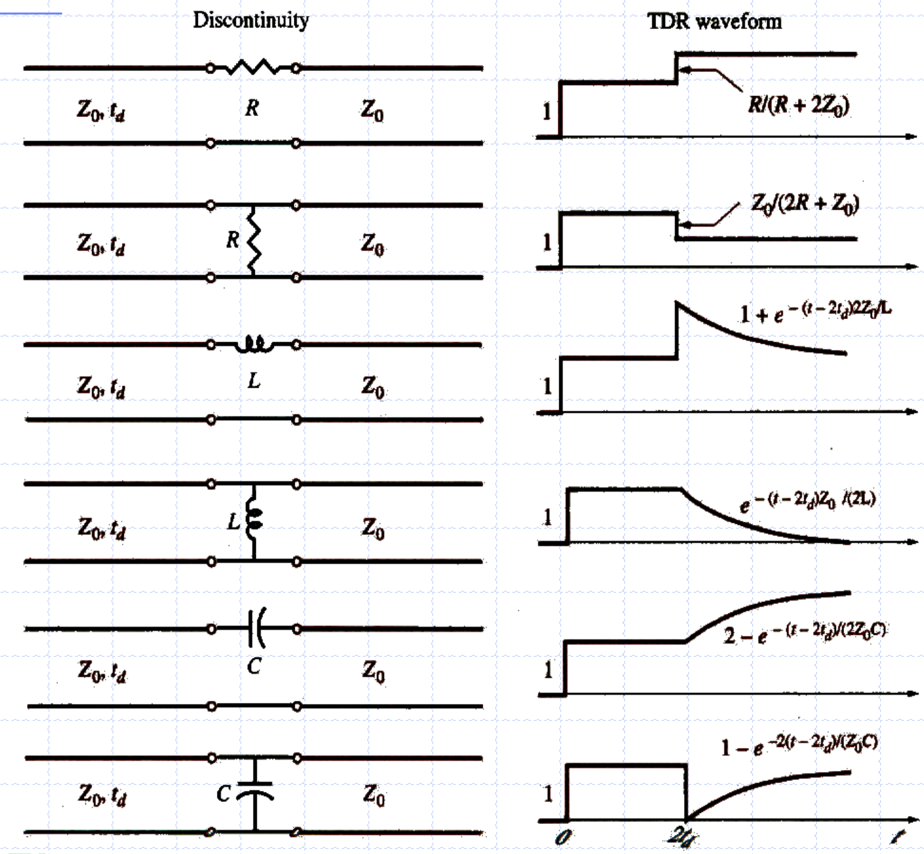




(a)



TDR signatures produced by simple discontinuities



6.5 Line with Initial Condition

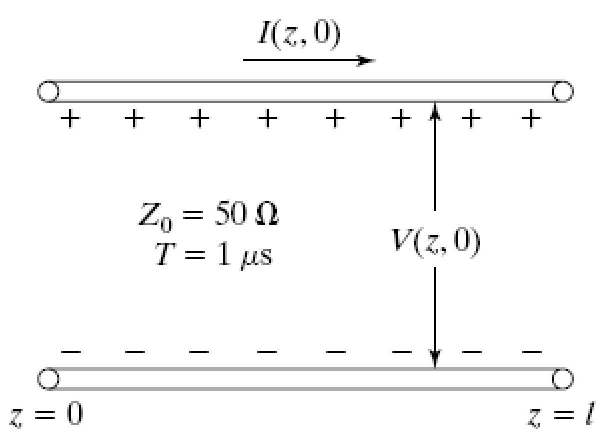
Arbitrary initial distribution

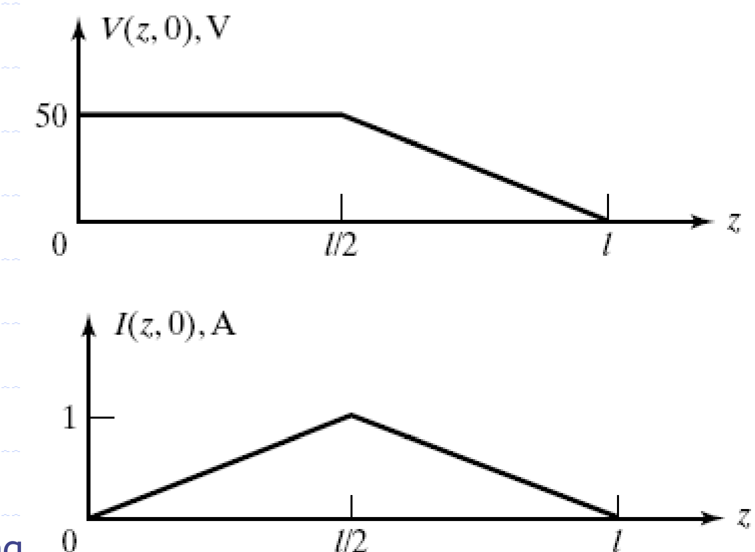
Thus far, we have considered lines with quiescent initial conditions, that is, with no initial voltages and currents on them.

As a prelude to the discussion of analysis of interconnections between logic gates, we shall now consider lines with nonzero initial conditions.

We discuss first the general case of arbitrary initial voltage and current distributions by decomposing them into (+) and (-) wave voltages and currents.

A line open-circuited at both ends is charged initially, say, at to the voltage and current distributions shown in the figure. What are the decomposed (+) and (-) waves?





6.5 Line with Initial Condition

Arbitrary initial distribution

Writing the line voltage and current distributions as sums of (+) and (-) wave voltages and currents

$$V^+(z,0)+V^-(z,0)=V(z,0)$$

$$I^+(z,0)+I^-(z,0)=I(z,0)$$

$$\downarrow$$

$$V^+(z,0)-V^-(z,0)=Z_0 I(z,0)$$

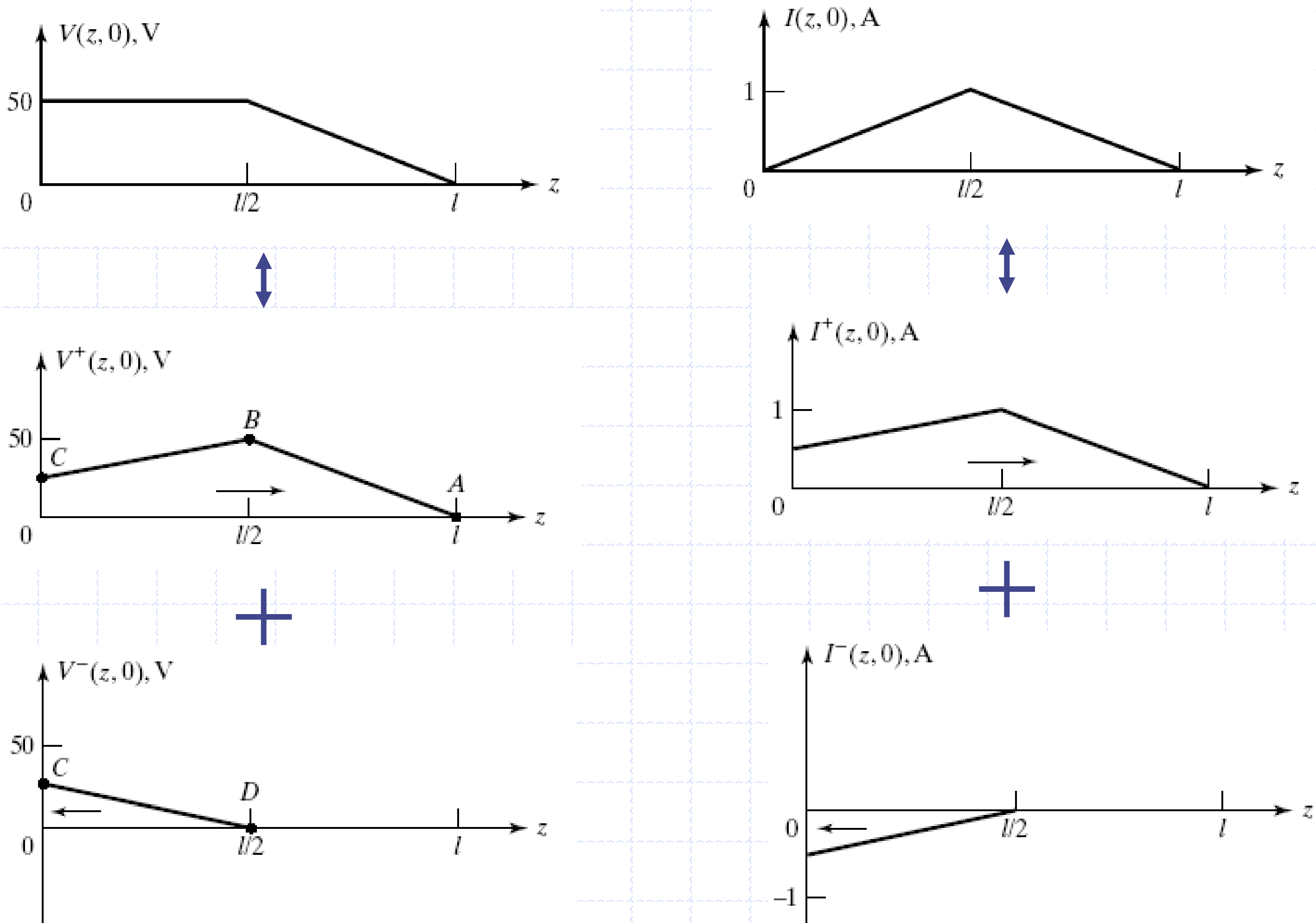
$$V^+(z,0)=\frac{1}{2}\left[V(z,0)+Z_0 I(z,0)\right]$$

$$V^-(z,0)=\frac{1}{2}\left[V(z,0)-Z_0 I(z,0)\right]$$

6.5 Line with Initial Condition

Arbitrary initial distribution

t = 0



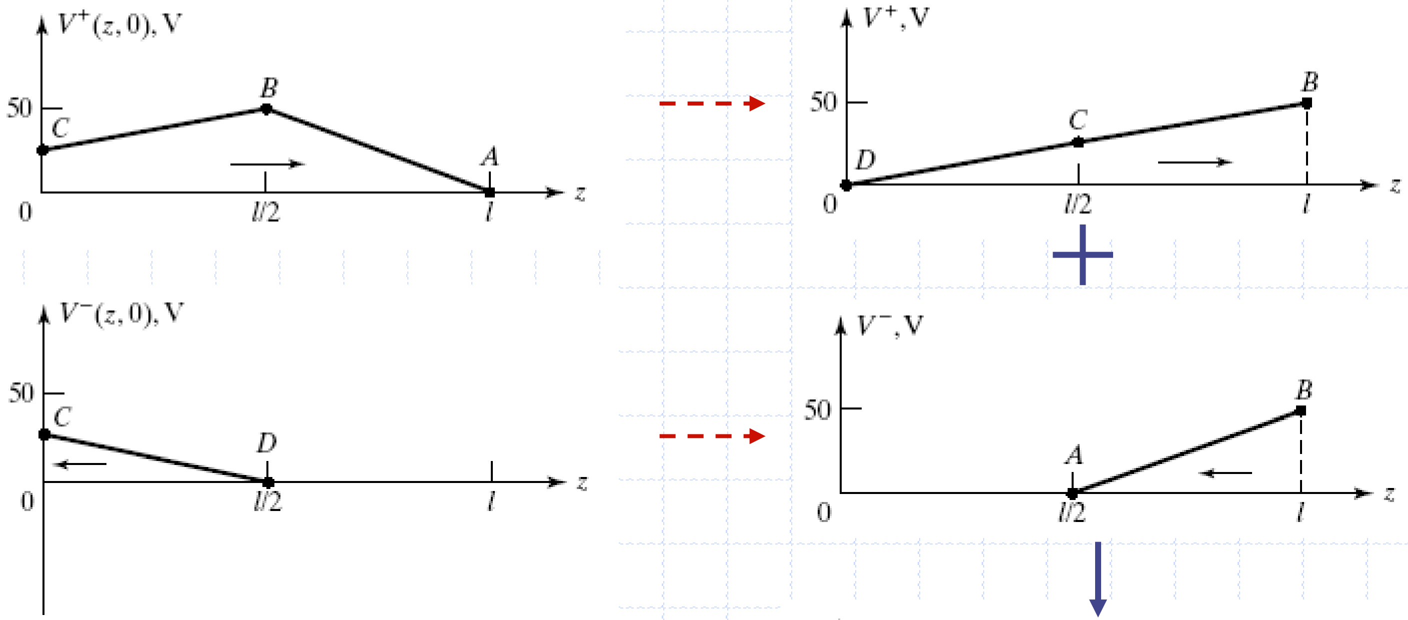
Can you plot the (+) and (-) waveform at t = 0.5us?

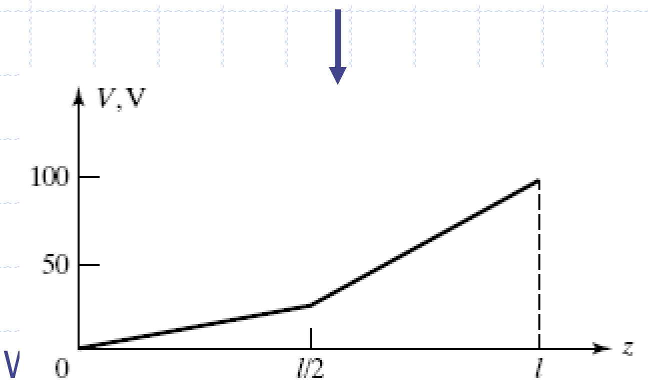
6.5 Line with Initial Condition

Arbitrary initial distribution

Key concept: voltage reflection coefficient of 1 and current reflection coefficient of -1 at both ends.

voltage

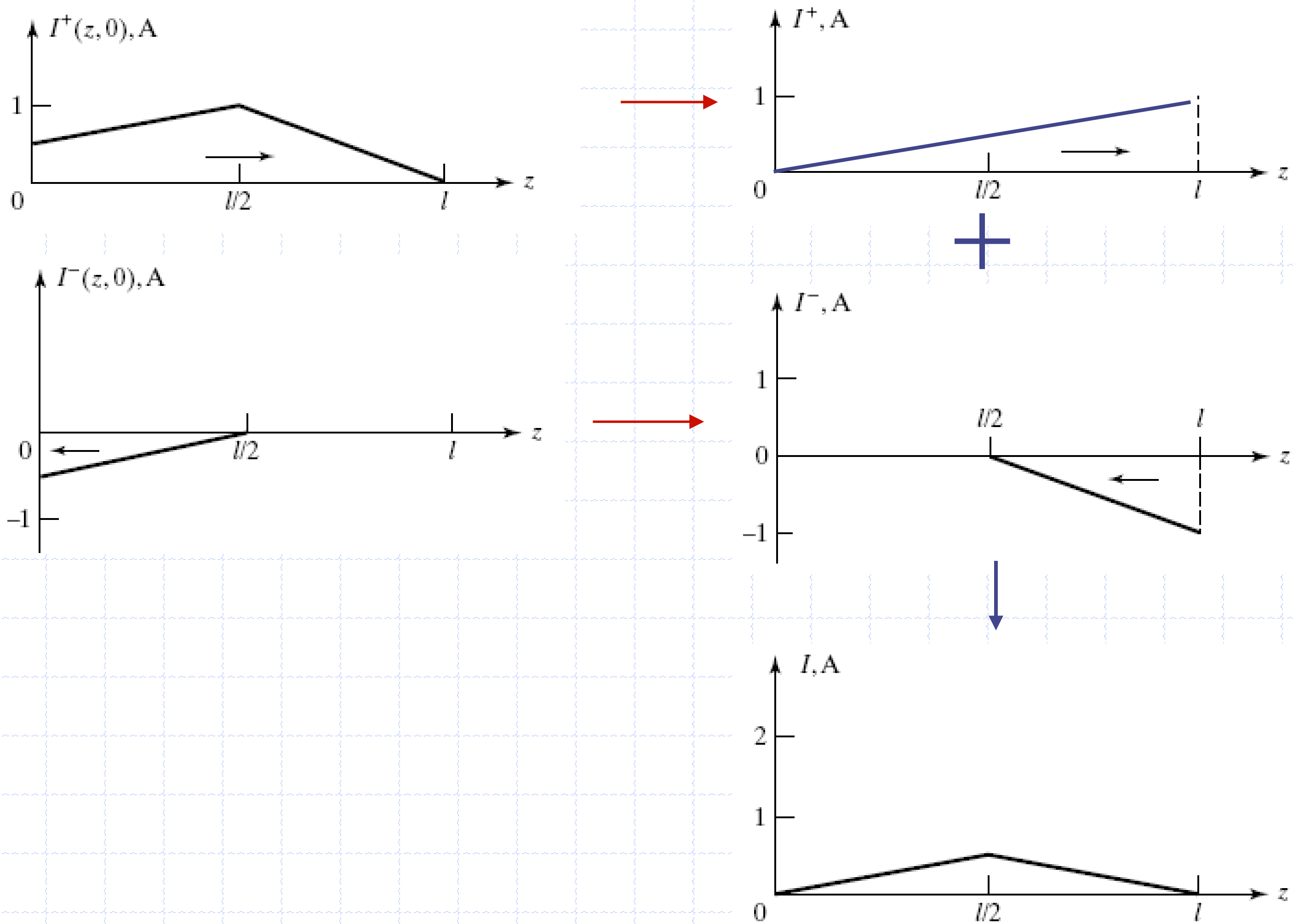




6.5 Line with Initial Condition

Arbitrary initial distribution

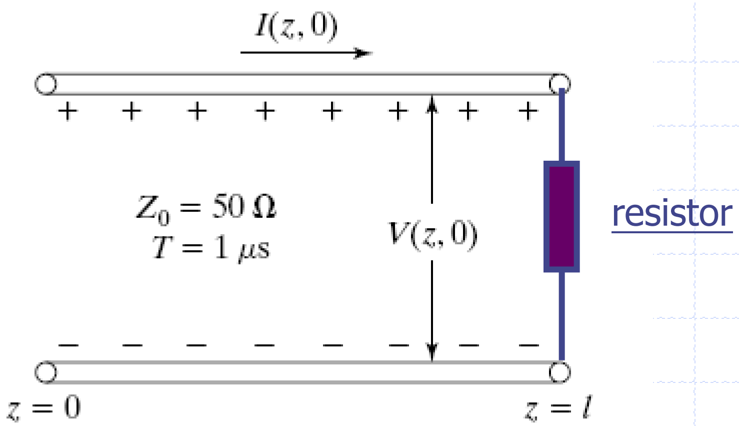
current

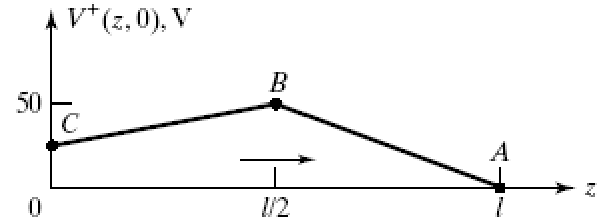


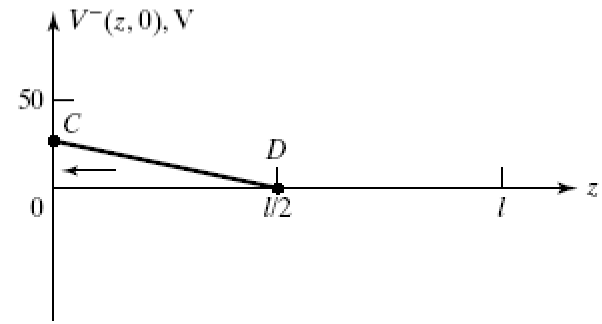
6.5 Line with Initial Condition

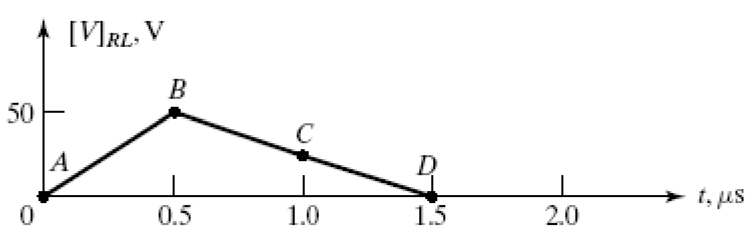
Arbitrary initial distribution

What happen as you put a resistor load at z = *l* and t = 0 ?







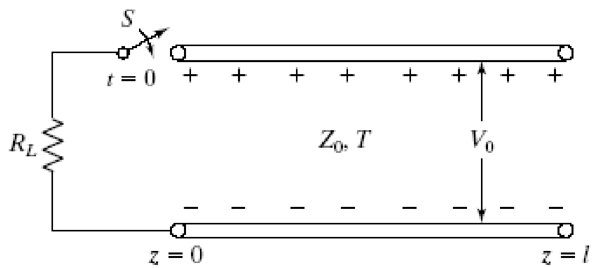


6.5 Line with Initial Condition

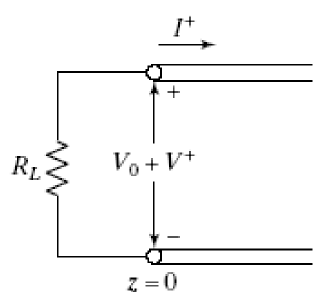
uniform initial distribution (ex: 6.6)

The analysis can be carried out with the aid of superposition and bounce diagrams.

Let us consider a line of $Z_0 = 50\ \Omega$ and $T = 1\ \mu s$ initially charged to uniform voltage $V_0 = 100$ V and zero current. A resistor $R_L = 150\ \Omega$ is connected at $t = 0$ to the end $z = 0$ of the line, as shown in Fig. 6.43(a). We wish to obtain the time variation of the voltage across R_L for $t > 0$.



What happened as the switch is on at t = 0 ?
Can you plot the voltage v.s. time on the R_L?



$$V_0 + V^+ = -R_L I^+$$

$$V_0 + V^+ = -\frac{R_L}{Z_0} V^+$$

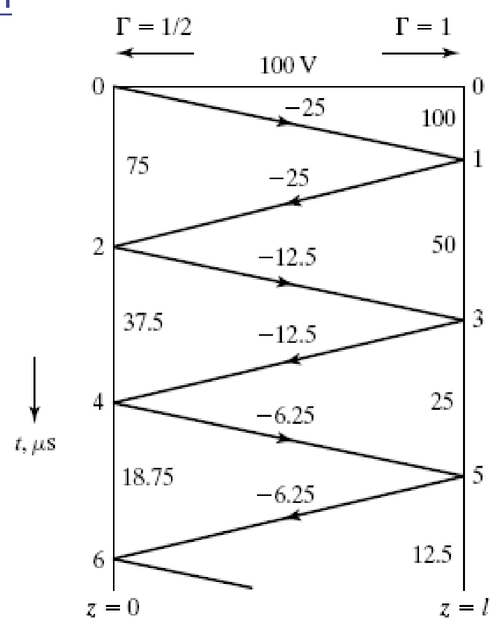
$$V^+ = -V_0 \frac{Z_0}{R_L + Z_0}$$

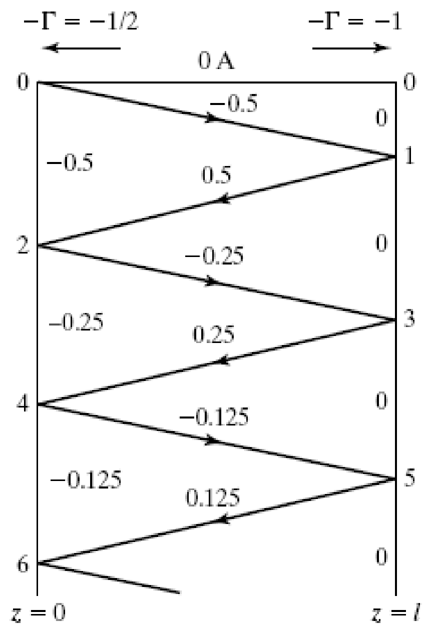
$$I^+ = -V_0 \frac{1}{R_L + Z_0}$$

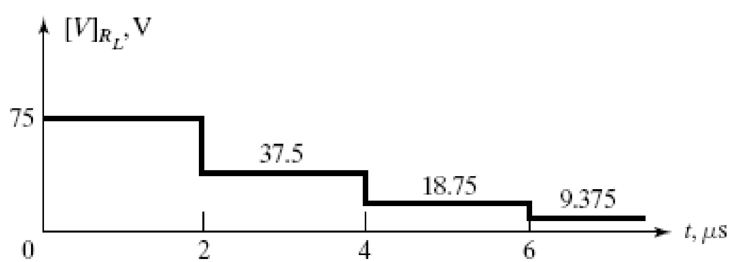
6.5 Line with Initial Condition

uniform initial distribution (ex: 6.6)

- Bounce diagram







Is the energy balanced between t = 0- and t = 0+?

What is the stored energy at t = 0- ?

Where does the energy go at t > 0 ?

6.5 Line with Initial Condition

uniform initial distribution (ex: 6.6)

At t = 0-, energy is stored in both electric and magnetic fields in the line, with energy densities ½ C V^2 and ½ L I^2 respectively.

$$W_e = \frac{1}{2} C V_0^{\,2} l = \frac{1}{2} C V_0^{\,2} v_p T = \frac{1}{2} C V_0^{\,2} \frac{1}{\sqrt{LC}} T = \frac{1}{2} \frac{V_0^2}{Z_0} T = 10^{-4} J$$

$$W_m = \frac{1}{2} L I_0^{\,2} l = \frac{1}{2} L I_0^{\,2} v_p T = \frac{1}{2} L I_0^{\,2} \frac{1}{\sqrt{LC}} T = \frac{1}{2} I_0^2 Z_0 T = 0$$

$$(V_0 = 100V, I_0 = 0, T = 1\mu s)$$

After t = 0,

$$W_d = \int_{t=0}^{\infty} P_d \, dt = \int_0^{2\times10^{-6}} \frac{75^2}{150} dt + \int_{2\times10^{-6}}^{4\times10^{-6}} \frac{37.5^2}{150} dt + \int_{4\times10^{-6}}^{6\times10^{-6}} \frac{18.75^2}{150} dt + \cdots = \frac{2\times10^{-6}}{150} \times 75^2 \left(1 + \frac{1}{4} + \frac{1}{16} + \cdots\right) = 10^{-4}\,\mathrm{J}$$

Power is conservative!

6.6 Interconnects between Logic Gates

Load-line technique

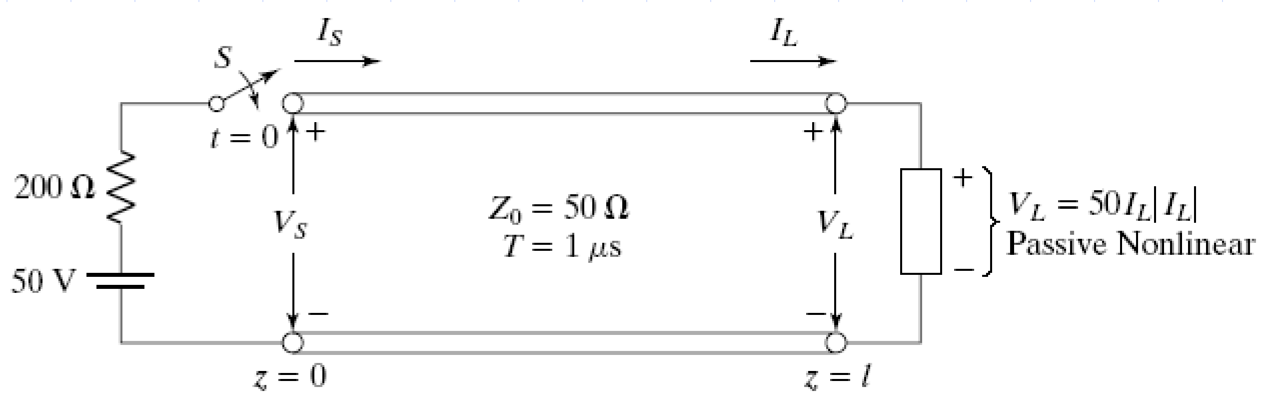
Thus far we have been concerned with time-domain analysis for lines with terminations and discontinuities made up of linear circuit elements.

Logic gates present nonlinear resistive terminations to the interconnecting transmission lines in digital circuits.

The analysis is then made convenient by a graphical technique known as the *load-line* technique.

Ex: 6.7

We wish to obtain the time variations of the voltages V_s and V_L at the source and load ends, respectively, following the closure of the switch *S at t =0.*



6.6 Interconnects between Logic Gates

Load-line technique

t = 0+ and z = 0:

$$50 = 200 I_S + V_S$$

$$\boxed{\begin{aligned} &V_S = V^+ \\ &I_S = I^+ = \frac{V^+}{Z_0} = \frac{V_S}{50} \end{aligned}}$$

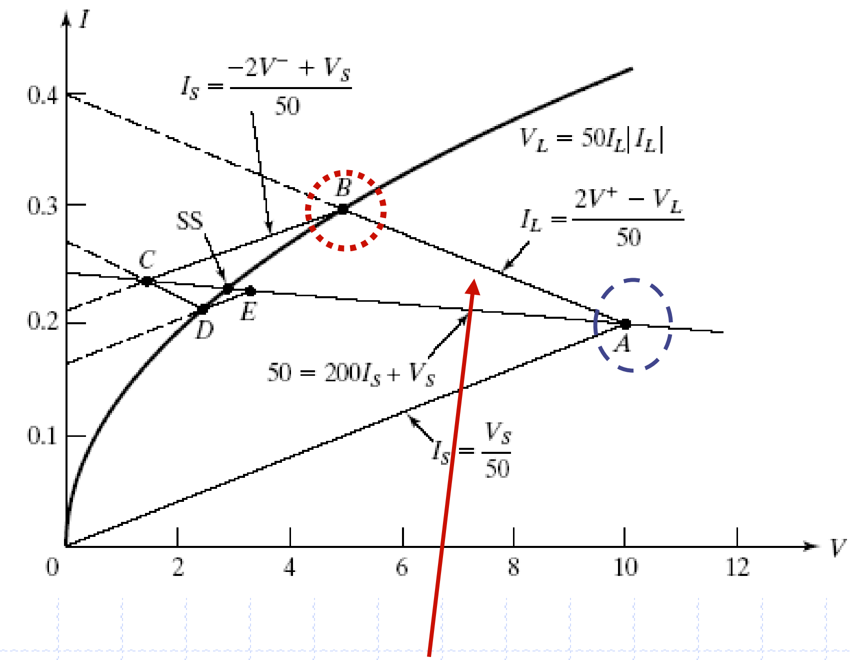
When the (+) wave reaches the load end at z = l and t = T

$$\boxed{V_L = 50 I_L \, | I_L |}$$

$$V_L = V^+ + V^-$$

$$I_L = I^+ + I^- = \frac{V^+ - V^-}{Z_0}$$

$$= \frac{V^+ - (V_L - V^+)}{50} = \frac{2V^+ - V_L}{50}$$



Why does the line cross the point A?

6.6 Interconnects between Logic Gates

Load-line technique

Noting in the equation $I_L = (2V^{+} - V_L)/50$

Setting $V_L = V^{+}$

Then $I_L = I^{+}/50$

Thus, the point A is also on this line

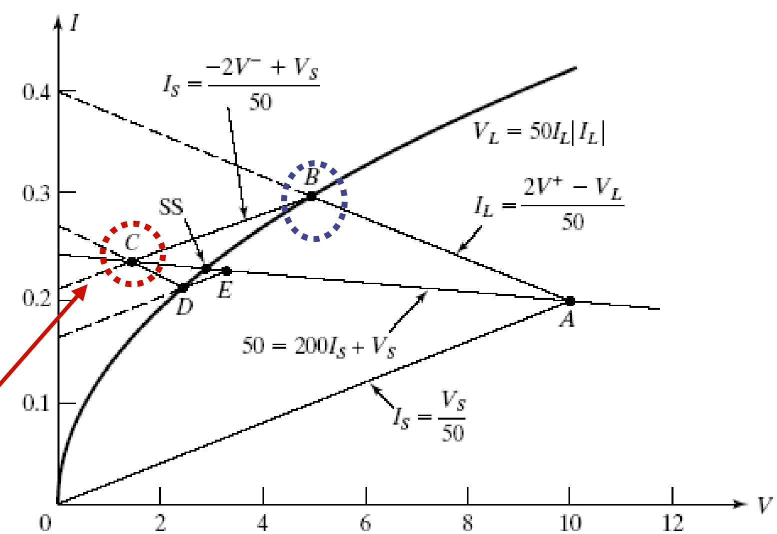
When the (-) wave reaches the source end at z = 0 and t = 2T, it sets up a reflection.

$$\boxed{50 = 200 I_S + V_S}$$

$$V_S = V^{+} + V^{-} + V^{-+}$$

$$I_S = I^{+} + I^{-} + I^{-+} = \frac{V^{+} - V^{-} + V^{-+}}{Z_0}$$

$$= \frac{V^{+} - V^{-} + \left(V_S - V^{+} - V^{-}\right)}{50} = \frac{-2V^{-} + V_S}{50}$$



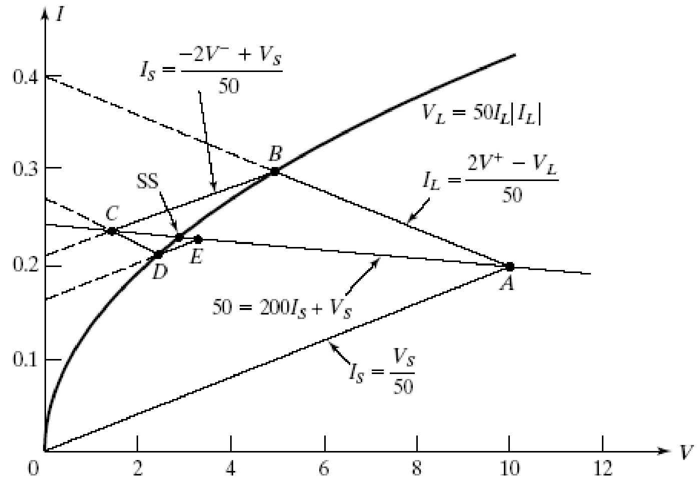
Why does the line cross the point B?

6.6 Interconnects between Logic Gates

Load-line technique

Note that $I_s = (-2V^- + V_s)/50$

For $V_s = V^+ + V^-$
We can obtain $I_s = (V^+ - V^-)/50$
Thus, this line crosses point B.

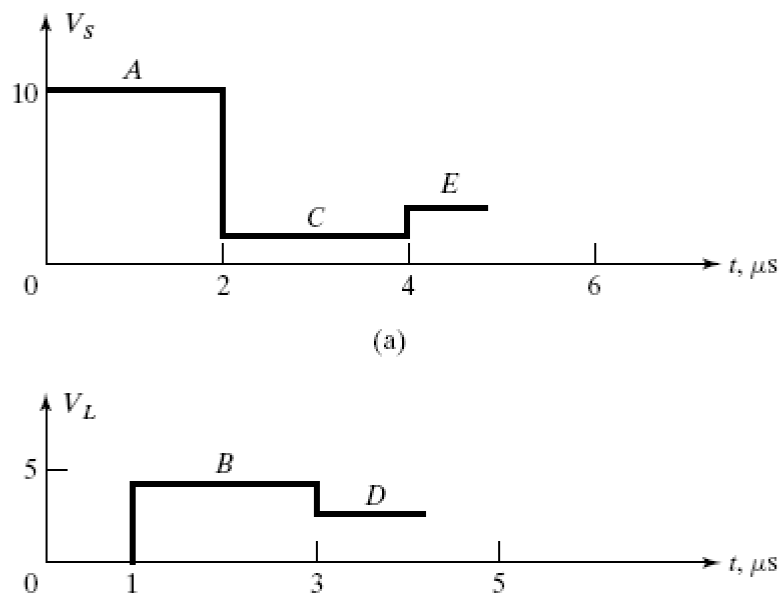


Continuing in this manner, we observe that the solution consists of obtaining the points of intersection on the source and load *V-I* characteristics by drawing successively straight lines of slope $1/Z_0$ and $-1/Z_0$ and beginning at the origin (the initial state) and with each straight line originating at the previous point of intersection.

6.6 Interconnects between Logic Gates

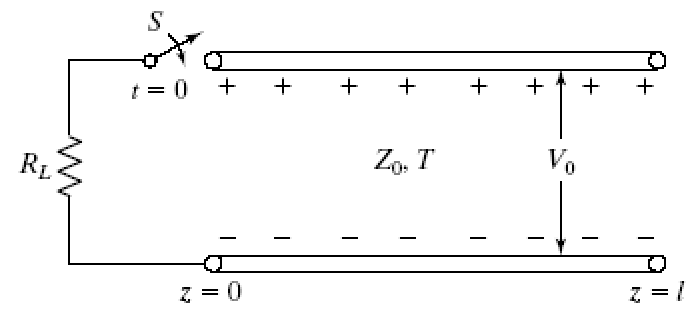
Load-line technique

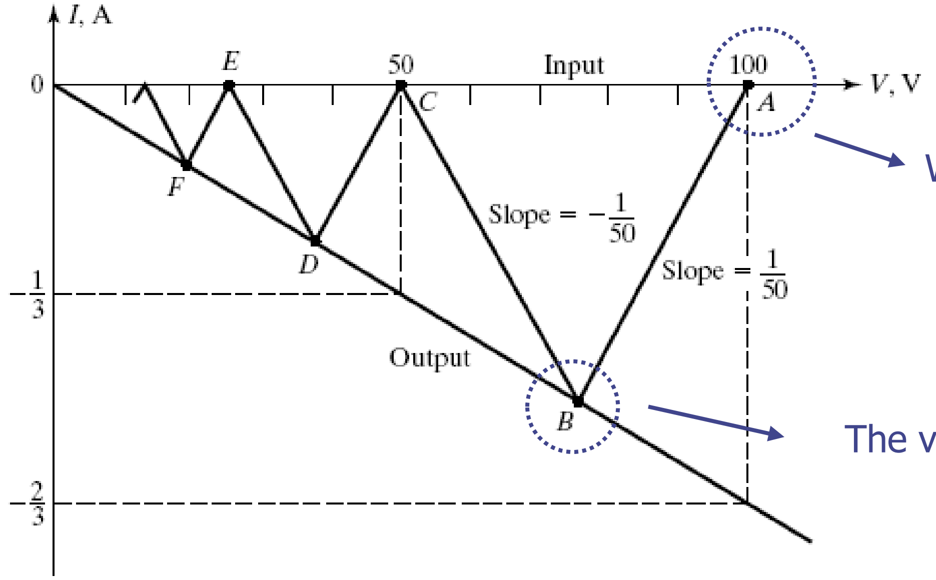
Output waveform at source and load



6.6 Interconnects between Logic Gates

Load-line technique – uniformed charged line





Why start here?

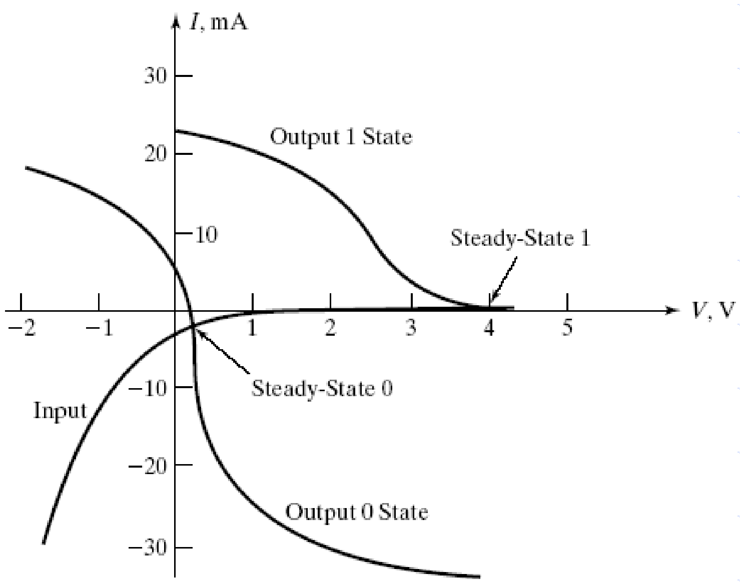
The voltage at which side?

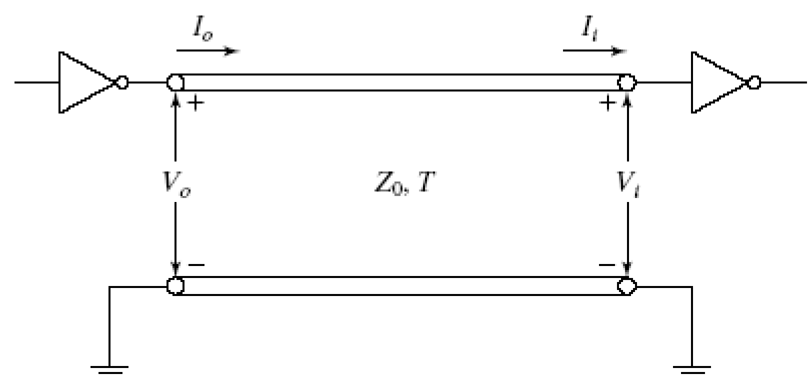
6.6 Interconnects between Logic Gates

Load-line technique – *Interconnection between logic gates*

Two transistor-transistor logic (TTL) inverters are interconnected by using a transmission line of characteristic impedance and one-way travel time *T*.

We wish to study the <u>transient phenomena</u> corresponding to the transition when the output of the first gate switches from the 0 to the 1 state, and vice versa.

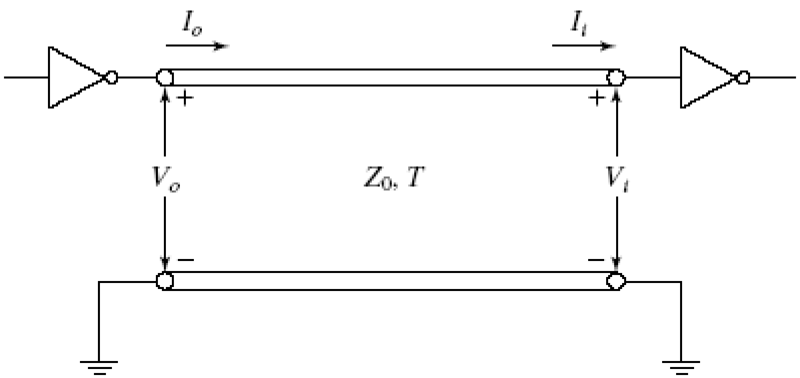


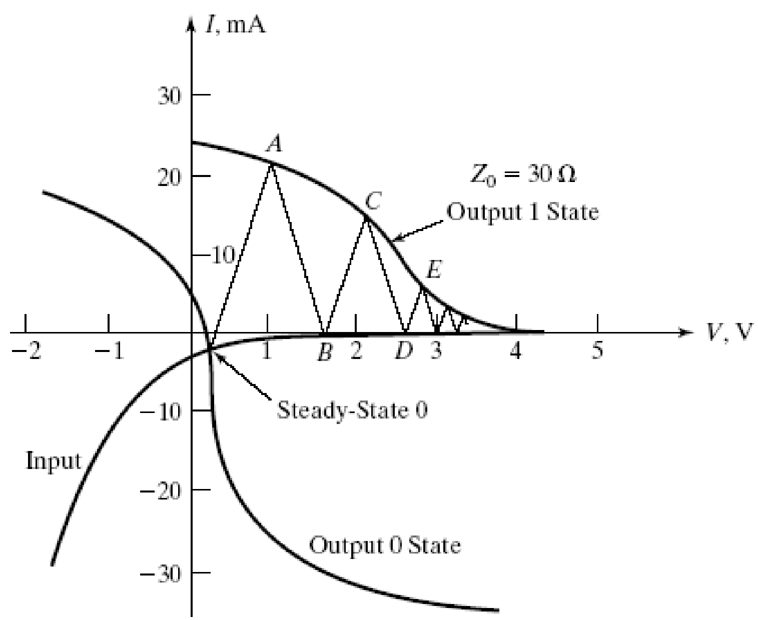


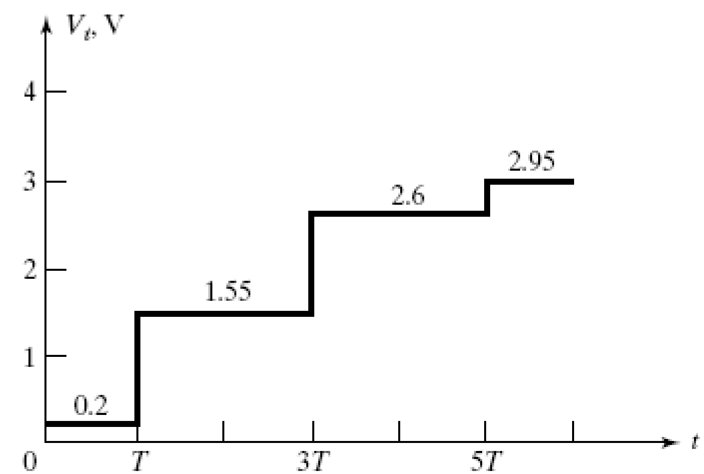
6.6 Interconnects between Logic Gates

Load-line technique – *Interconnection between logic gates*

Analysis of 0-to-1 transition



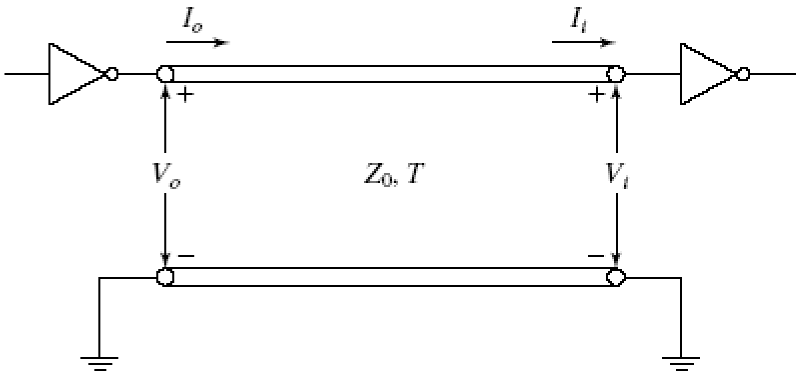


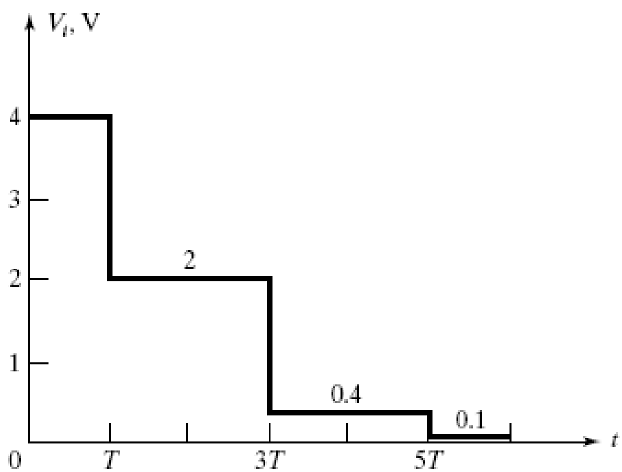


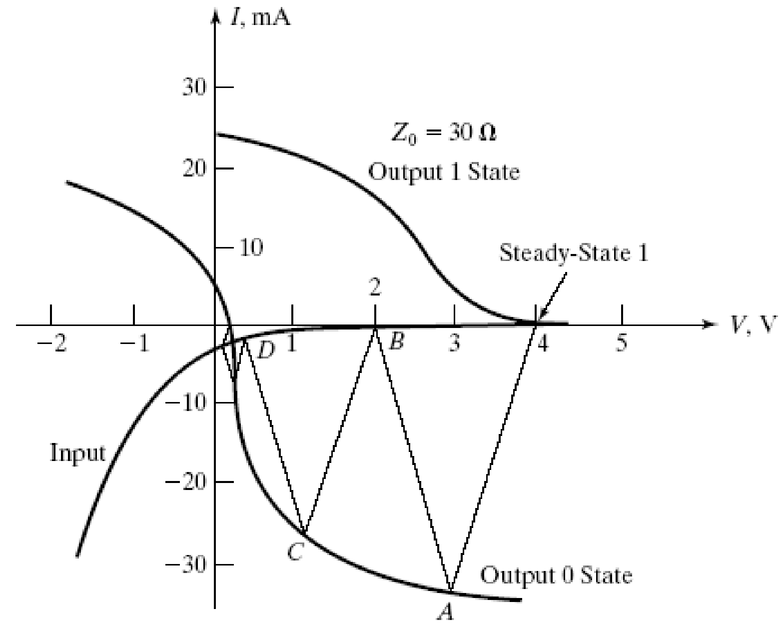
6.6 Interconnects between Logic Gates

Load-line technique – *Interconnection between logic gates*

Analysis of 1-to-0 transition







6.7 Crosstalk on Transmission Line

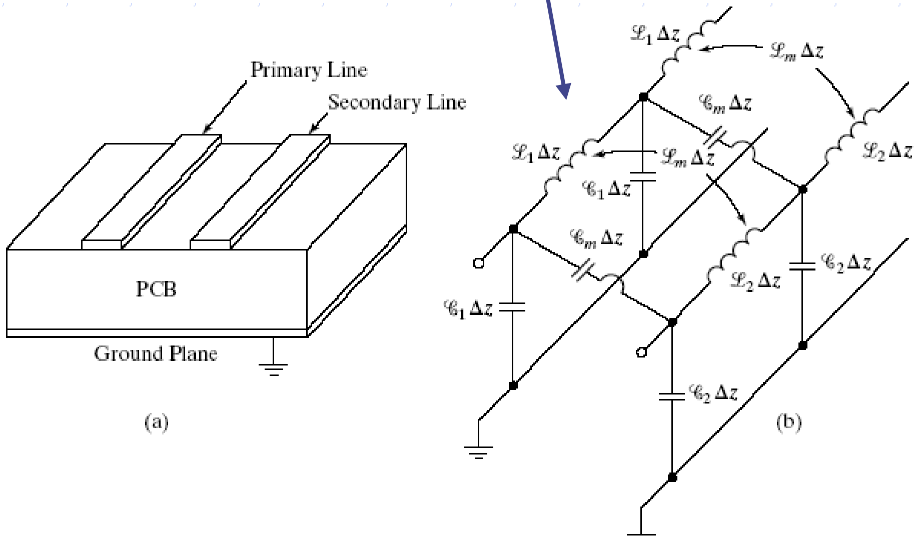
crosstalk mechanism and assumption

We shall analyze a pair of coupled transmission lines for the determination of induced waves on the secondary line for a given wave on the primary line.

To keep the analysis simple, we shall consider both lines to be of the same characteristic impedance, velocity of propagation, and length, and terminated by their characteristic impedances, so that no reflections occur from the ends of either line.

It is also convenient to assume the coupling to be weak, so that the effects on the primary line of waves induced in the secondary line can be neglected.

Capacitive coupling (electric field) : Cm
Inductive coupling (magnetic field) : Lm



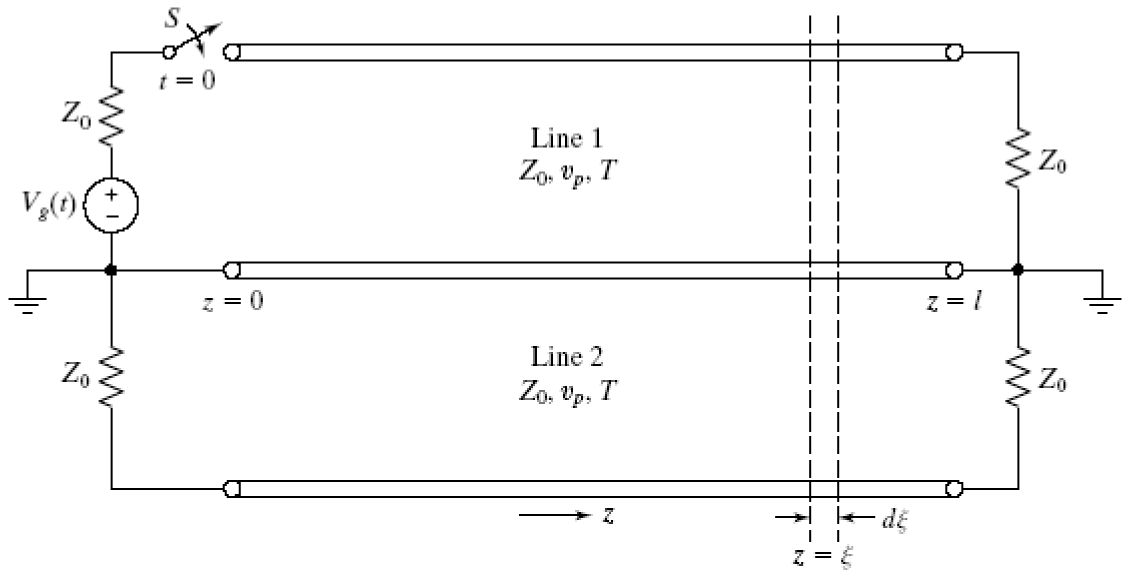
6.7 Crosstalk on Transmission Line

weak coupling analysis

As the wave propagates on the primary line from source toward load, each infinitesimal length of that line induces voltage and current in the adjacent infinitesimal length of the secondary line, which set up (+) and (-) waves on that line.

The contributions due to the infinitesimal lengths add up to give the induced voltage and current at a given location on the secondary line

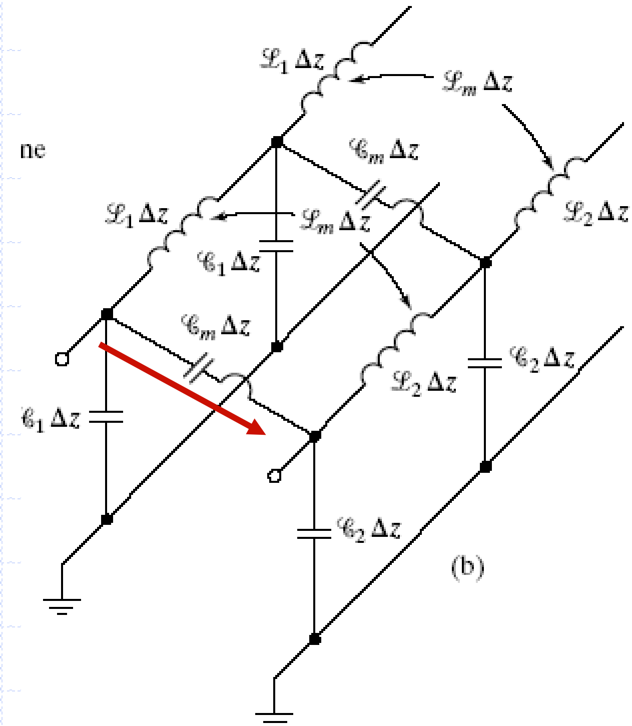
When the switch *S* is closed at t = 0, a (+) wave originates at z = 0 on line 1 and propagates toward the load.
Let us consider a differential length dξ at the location z = ξ of line 1 charged to the (+) wave voltage and current.
We try to obtain its contributions to the induced voltages and currents in line 2.

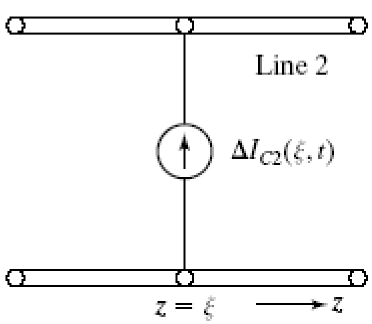


6.7 Crosstalk on Transmission Line

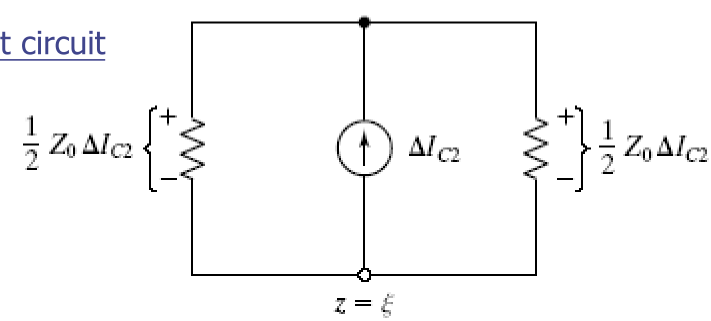
modeling for capacitive coupling

$$\Delta I_{c2}(\xi,t) = C_m \Delta\xi \frac{\partial V_1(\xi,t)}{\partial t}$$



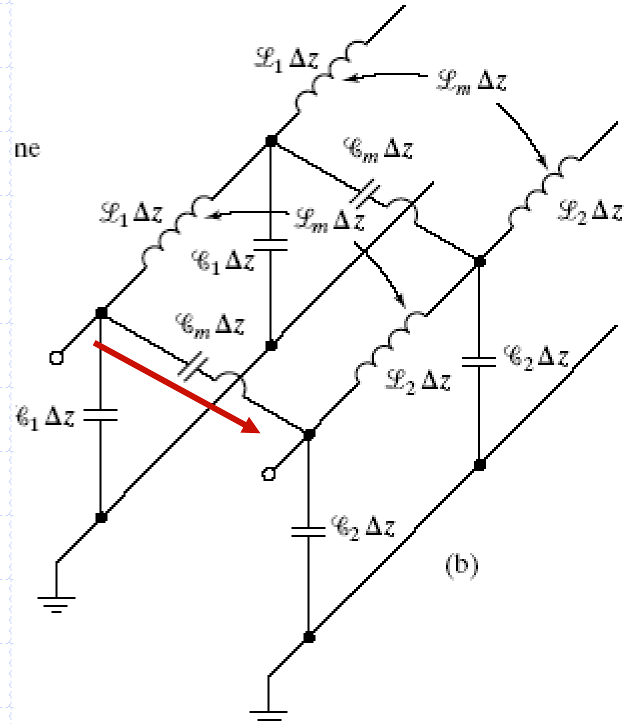


Equivalent circuit

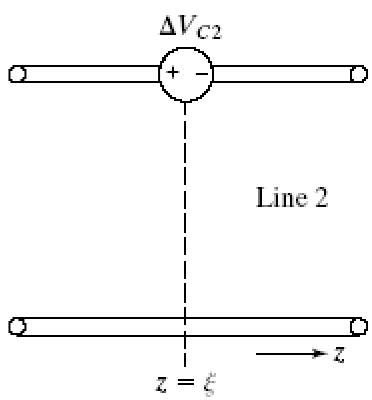


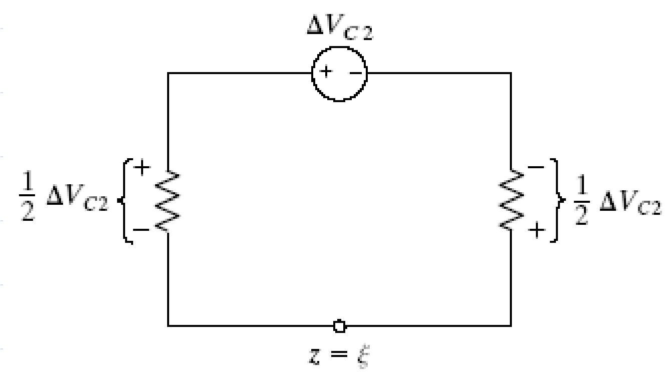
6.7 Crosstalk on Transmission Line

modeling for inductive coupling



$$\Delta V_{c2}(\xi, t) = \mathrm{L}_m \Delta \xi \frac{\partial I_1(\xi, t)}{\partial t}$$





6.7 Crosstalk on Transmission Line

modeling for inductive coupling

Combining the contributions due to capacitive coupling and inductive coupling

$$\Delta V_2^+ = \frac{1}{2} Z_0 \Delta I_{c2} - \frac{1}{2} \Delta V_{c2}$$

$$\Delta V_2^- = \frac{1}{2} Z_0 \Delta I_{c2} + \frac{1}{2} \Delta V_{c2}$$

$$\Delta V_2^+(\xi, t) = \left[\frac{1}{2} \mathrm{C}_m Z_0 \frac{\partial V_1(\xi, t)}{\partial t} - \frac{1}{2} \mathrm{L}_m \frac{\partial I_1(\xi, t)}{\partial t} \right] \Delta \xi$$

$$= \frac{1}{2} \left(\mathrm{C}_m Z_0 - \frac{\mathrm{L}_m}{Z_0} \right) \frac{\partial V_1(\xi, t)}{\partial t} \Delta \xi$$

$$\Delta V_2^-(\xi, t) = \frac{1}{2} \left(\mathrm{C}_m Z_0 + \frac{\mathrm{L}_m}{Z_0} \right) \frac{\partial V_1(\xi, t)}{\partial t} \Delta \xi$$

6.7 Crosstalk on Transmission Line

forward crosstalk voltage and current

Noting that the effect of V_1 at $z = \xi$ at a given time *t* is felt at a location $z > \xi$ on line 2 at time $t + (z - \xi) / v_p$, we can write

$$\Delta V_2^+(z,t) = \int_0^z \frac{1}{2}\left(C_m Z_0 - \frac{L_m}{Z_0}\right)\frac{\partial}{\partial t}\left[V_1\left(t - \frac{\xi}{v_p} - \frac{z-\xi}{v_p}\right)\right]d\xi$$

$$= \frac{1}{2}\left(C_m Z_0 - \frac{L_m}{Z_0}\right)\int_0^z \frac{\partial V_1\left(t - z/v_p\right)}{\partial t}d\xi$$

$$V_2^+(z,t) = zK_f V_1^{'}(t - z/v_p)$$

$$K_f = \frac{1}{2}\left(C_m Z_0 - \frac{L_m}{Z_0}\right)$$

forward-crosstalk coefficient

Note that the upper limit in the integral is *z*, because the line-1 voltage to the right of a given location *z* on that line does not contribute to the forward-crosstalk voltage on line 2 at that same location.

6.7 Crosstalk on Transmission Line

backward crosstalk voltage and current

Noting that the effect of V_1 at $z = \xi$ at a given time *t* is felt at a location $z < \xi$ on line 2 at time $t + (\xi - z) / v_p$.

$$V_2^-(z,t) = \int_z^l \frac{1}{2}\left(C_m Z_0 + \frac{L_m}{Z_0}\right)\frac{\partial}{\partial t}\left[V_1\left(t - \frac{\xi}{v_p} - \frac{\xi - z}{v_p}\right)\right]d\xi$$

$$= \frac{1}{2}\left(C_m Z_0 + \frac{L_m}{Z_0}\right)\int_z^l \frac{\partial}{\partial t}\left[V_1\left(t + \frac{z}{v_p} - \frac{2\xi}{v_p}\right)\right]d\xi$$

$$= -\frac{1}{4}v_p\left(C_m Z_0 + \frac{L_m}{Z_0}\right)\int_z^l \frac{\partial}{\partial \xi}\left[V_1\left(t + \frac{z}{v_p} - \frac{2\xi}{v_p}\right)\right]d\xi$$

$$= -\frac{1}{4}v_p\left(C_m Z_0 + \frac{L_m}{Z_0}\right)\left[V_1\left(t + \frac{z}{v_p} - \frac{2\xi}{v_p}\right)\right]_{\xi=z}^{l}$$

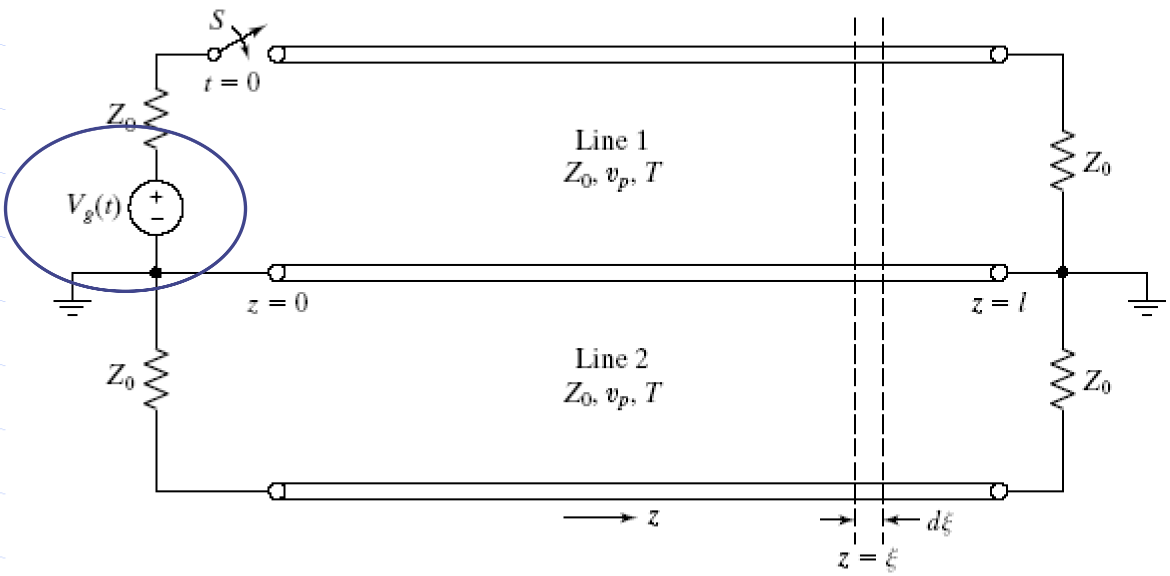
$$V_2^-(z,t) = K_b\left[V_1\left(t - \frac{z}{v_p}\right) - V_1\left(t - \frac{2l}{v_p} + \frac{z}{v_p}\right)\right]$$

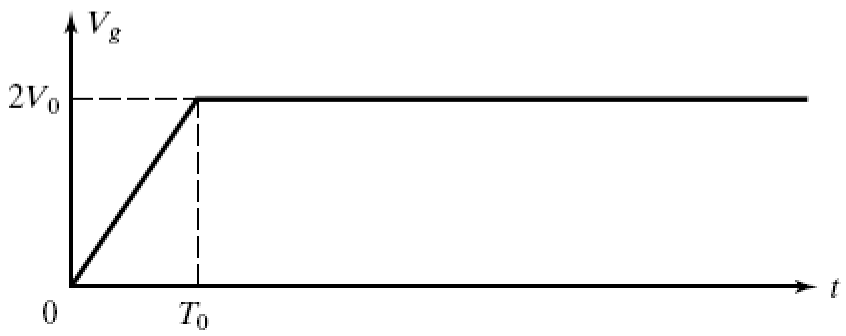
$$K_b = \frac{1}{4}v_p\left(C_m Z_0 + \frac{L_m}{Z_0}\right)$$

backward-crosstalk coefficient

The lower limit in the integral is *z*, because the line-1 voltage to the left of a given location *z* on that line does not contribute to the backward crosstalk voltage on line 2 at that same location.

Ex: Determination of induced wave voltages in the secondary line of a coupled pair of lines





Ex: Determination of induced wave voltages in the secondary line of a coupled pair of lines

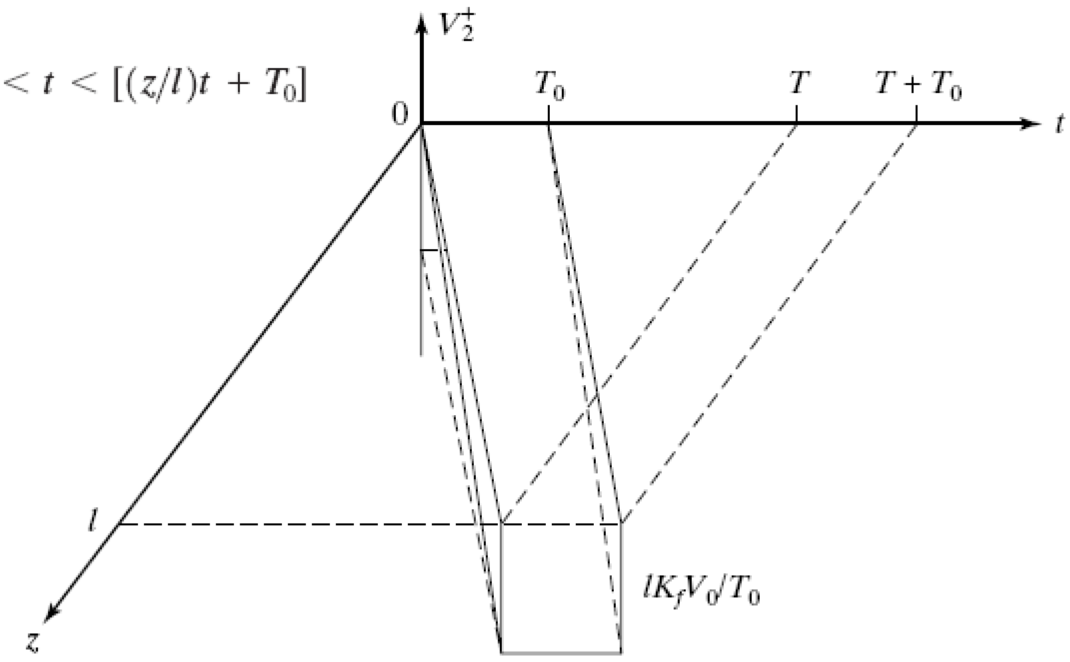
$$V_2^+(z,t) = zK_f V_1'(t - z/v_p)$$

$$= \begin{cases} zK_f V_0/T_0 & \text{for} \quad 0 < (t - z/v_p) < T_0 \\ 0 & \text{otherwise} \end{cases}$$

$$= \begin{cases} zK_f V_0/T_0 & \text{for} \quad (z/v_p) < t < (z/v_p + T_0) \\ 0 & \text{otherwise} \end{cases}$$

$$= \begin{cases} zK_f V_0/T_0 & \text{for} \quad (z/l)T < t < [(z/l)t + T_0] \\ 0 & \text{otherwise} \end{cases}$$

Far end



Ex: Determination of induced wave voltages in the secondary line of a coupled pair of lines

Near End

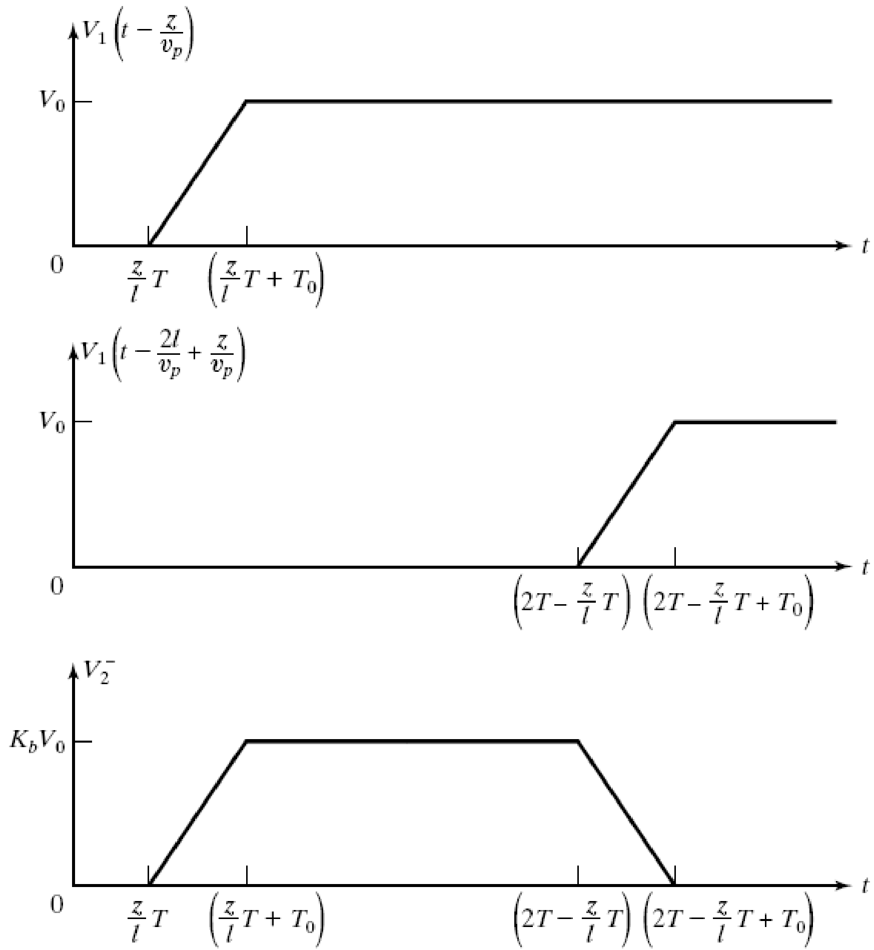
$$V_2^-(z,t) = K_b[V_1(t - z/v_p) - V_1(t - 2l/v_p + z/v_p)]$$

$$V_1\left(t - \frac{z}{v_p}\right) = \begin{cases} \frac{V_0}{T_0}\left(t - \frac{z}{v_p}\right) & \text{for } 0 < \left(t - \frac{z}{v_p}\right) < T_0 \\ 0 & \text{for } \left(t - \frac{z}{v_p}\right) > T_0 \end{cases}$$

$$= \begin{cases} \frac{V_0}{T_0}\left(t - \frac{z}{l}T\right) & \text{for } \frac{z}{l}T < t < \left(\frac{z}{l}T + T_0\right) \\ 0 & \text{for } t > \left(\frac{z}{l}T + T_0\right) \end{cases}$$

$$V_1\left(t - \frac{2l}{v_p} + \frac{z}{v_p}\right) = \begin{cases} \frac{V_0}{T_0}\left(t - \frac{2l}{v_p} + \frac{z}{v_p}\right) & \text{for } 0 < \left(t - \frac{2l}{v_p} + \frac{z}{v_p}\right) < T_0 \\ 0 & \text{for } \left(t - \frac{2l}{v_p} + \frac{z}{v_p}\right) > T_0 \end{cases}$$

$$= \begin{cases} \frac{V_0}{T_0}\left(t - 2T + \frac{z}{l}T\right) & \text{for } \left(2T - \frac{z}{l}T\right) < t < \left(2T - \frac{z}{l}T + T_0\right) \\ 0 & \text{for } t > \left(2T - \frac{z}{l}T + T_0\right) \end{cases}$$



Ex: Determination of induced wave voltages in the secondary line of a coupled pair of lines

Note that as z varies from zero to l, the shape of V_2^- changes from a trapezoidal pulse with a height of $K_b V_0$ at $z = 0$ to a triangular pulse of height $K_b V_0$ and width $2T_0$ at $z = (1 - T_0/2T)l$ and then changes to a trapezoidal pulse again but with a height continuously decreasing from $K_b V_0$ to zero at $z = l$.