# Microwave Filter Design

## Chp4. Transmission Lines and Components

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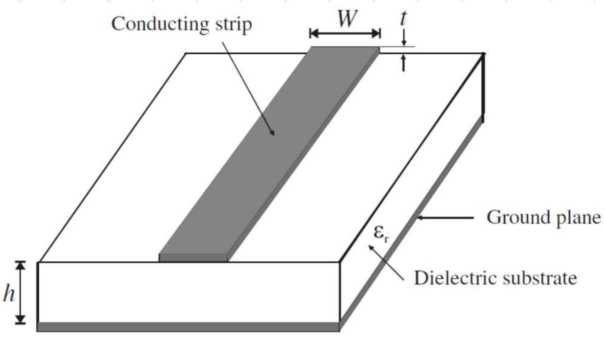
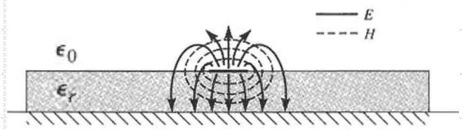
# Microstrip Lines

Microstrip Structure



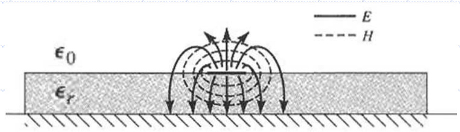



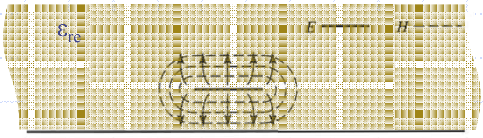

- Inhomogeneous structure:
  Due to the fields within two guided-wave media, the microstrip does not support a pure TEM wave.

- When the longitudinal components of the fields for the dominant mode of a microstrip line is much smaller than the transverse components, the quasi-TEM approximation is applicable to facilitate design.

# Microstrip Lines
# - Transmission Line Parameters

Effective Dielectric Constant ($\varepsilon_{re}$) and Characteristic Impedance($Z_C$)




- For thin conductors (i.e., t → 0), closed-form expression (error ≤ 1 % ):
  - W/h ≤ 1:

$$\varepsilon_{re} = \frac{\varepsilon_r + 1}{2} + \frac{\varepsilon_r - 1}{2}\left\{\left(1 + 12\frac{h}{W}\right)^{-0.5} + 0.04\left(1 - \frac{W}{h}\right)^2\right\}$$

$$Z_c = \frac{\eta}{2\pi\sqrt{\varepsilon_{re}}}\ln\left(\frac{8h}{W} + 0.25\frac{W}{h}\right)$$

  - W/h ≥ 1:

$$\varepsilon_{re} = \frac{\varepsilon_r + 1}{2} + \frac{\varepsilon_r - 1}{2}\left(1 + 12\frac{h}{W}\right)^{-0.5}$$

$$Z_c = \frac{\eta}{\sqrt{\varepsilon_{re}}}\left\{\frac{W}{h} + 1.393 + 0.677\ln\left(\frac{W}{h} + 1.444\right)\right\}^{-1}$$

- For thin conductors (i.e., t → 0), more accurate expressions:
  - Effective dielectric constant (error ≤ 0.2 % ):

$$\varepsilon_{re} = \frac{\varepsilon_r + 1}{2} + \frac{\varepsilon_r - 1}{2}\left(1 + \frac{10}{u}\right)^{-ab}$$

$$a = 1 + \frac{1}{49}\ln\left(\frac{u^4 + \left(\frac{u}{52}\right)^2}{u^4 + 0.432}\right) + \frac{1}{18.7}\ln\left[1 + \left(\frac{u}{18.1}\right)^3\right]$$

$$b = 0.564\left(\frac{\varepsilon_r - 0.9}{\varepsilon_r + 3}\right)^{0.053}$$

  - Characteristic impedance (error ≤ 0.03 % ):

$$Z_c = \frac{\eta}{2\pi\sqrt{\varepsilon_{re}}}\ln\left[\frac{F}{u} + \sqrt{1 + \left(\frac{2}{u}\right)^2}\right]$$

$$F = 6 + (2\pi - 6)\exp\left[-\left(\frac{30.666}{u}\right)^{0.7528}\right]$$



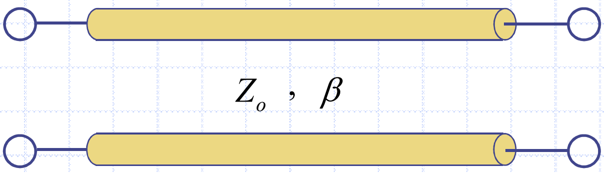
# Microstrip Lines
# - Transmission Line Parameters

- Guided wavelength

$$\lambda_g = \frac{\lambda_0}{\sqrt{\varepsilon_{re}}} \quad \text{or} \quad \lambda_g = \frac{300}{f(GHz)\sqrt{\varepsilon_{re}}}\,mm$$

- Propagation constant

$$\beta = \frac{2\pi}{\lambda_g}$$




- Phase velocity

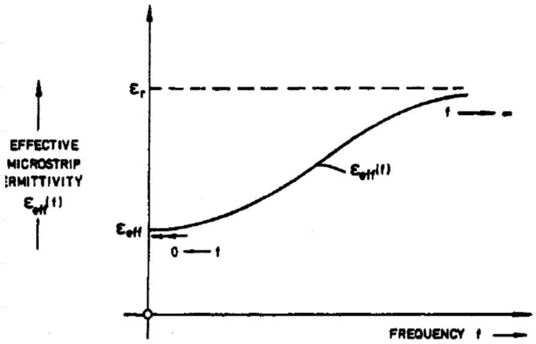
$$\upsilon_p = \frac{\omega}{\beta} = \frac{c}{\sqrt{\varepsilon_{re}}}$$

- Electrical length

$$\theta = \beta\ell$$

# Microstrip Lines
# - Transmission Line Parameters

- Losses
  - Conductor loss
  - Dielectric loss
  - Radiation loss
- Dispersion
  - $\varepsilon_{re}(f)$
  - $Z_o(f)$




- Surface Waves and higher-order modes
  - Coupling between the quasi-TEM mode and surface wave mode become significant when the frequency is above $f_s$

$$f_s = \frac{c \tan^{-1} \varepsilon_r}{\sqrt{2}\pi h \sqrt{\varepsilon_r - 1}}$$

  - Cutoff frequency $f_c$ of first higher-order modes in a microstrip

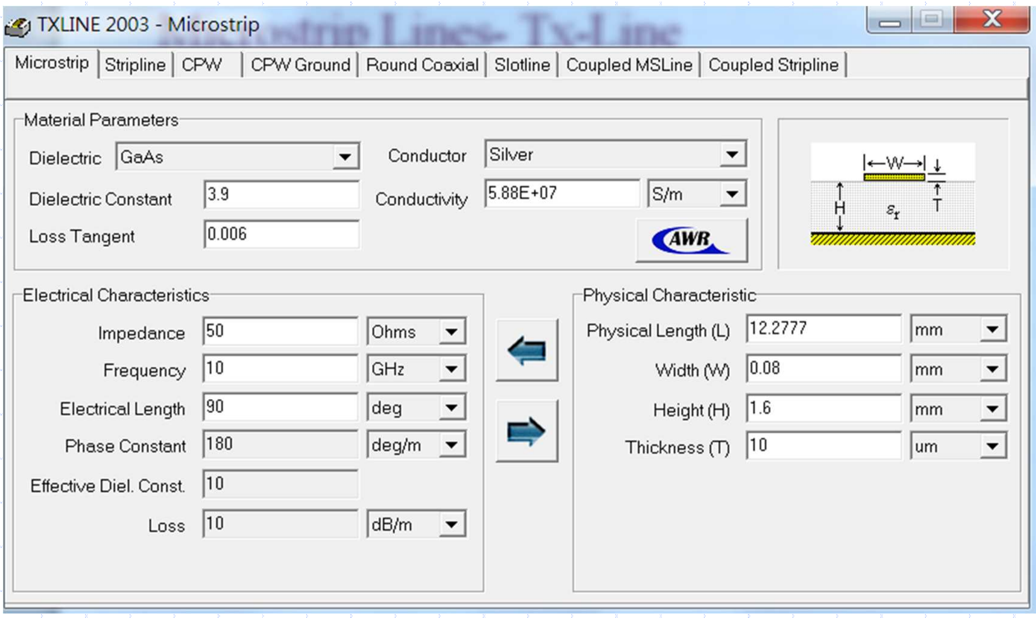
$$f_c = \frac{c}{\sqrt{\varepsilon_r}\,(2W + 0.8h)}$$

  - The operating frequency of a microstrip line < Min ($f_s$, $f_c$)



# Microstrip Lines
# - Tx-Line

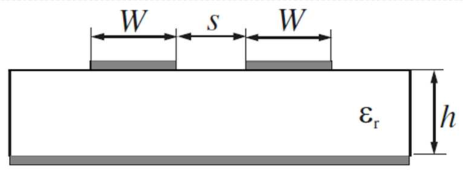
- Synthesis of transmission line – electrical or physical parameters



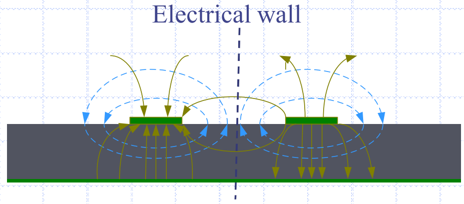
# Coupled Lines

Coupled line Structure

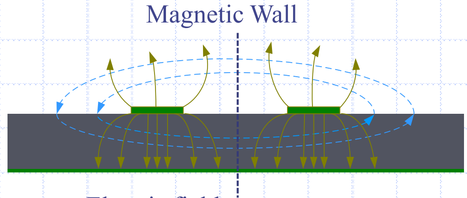
- The coupled line structure supports two quasi-TEM modes: odd mode and even mode.




Odd mode




Even mode

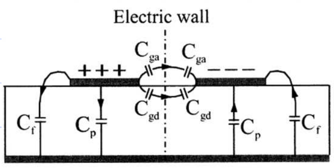




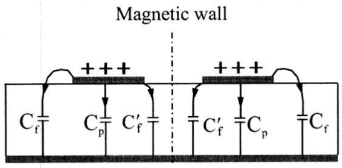

# Coupled Lines
# – Odd- and Even- Mode

Effective Dielectric Constant ($\varepsilon_{re}$) and Characteristic Impedance($Z_C$)

Odd mode




Even mode




- Odd- and Even- Mode:

  The characteristic impedances ($Z_{co}$ and $Z_{ce}$) and effective dielectric constants ($\varepsilon^o_{re}$ and $\varepsilon^e_{re}$) are obtained from the capacitances ($C_o$ and $C_e$):

  - Odd-Mode:

$$Z_{co} = (c\sqrt{C^a_o C_o})^{-1}$$

$$\varepsilon^o_{re} = C_o / C^a_o$$

  - Even-Mode:

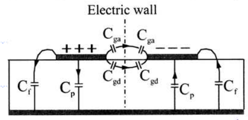
$$Z_{ce} = (c\sqrt{C^a_e C_e})^{-1}$$

$$\varepsilon^e_{re} = C_e / C^a_e$$

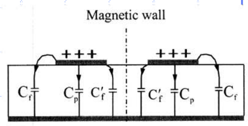
  - $C^a_o$ and $C^a_e$ are even- and odd-mode capacitances for the coupled microstrip line configuration with air as dielectric.

# Coupled Lines
# – Odd- and Even- Mode

Odd mode




Even mode




Effective Dielectric Constant ($\varepsilon_{re}$) and Characteristic Impedance($Z_C$)

- Odd- and Even- Mode Capacitances:
  - Odd-Mode:

    $$C_o = C_p + C_f + C_{gd} + C_{ga}$$

  - Even-Mode:

    $$C_e = C_p + C_f + C_f'$$

    - $C_p$ denotes the parallel plate capacitance between the strip and the ground plane:

      $$C_p = \varepsilon_o \varepsilon_r \, W/h$$

    - $C_f$ is the fringe capacitance as if for an uncoupled single microtrip line:

      $$2C_f = \sqrt{\varepsilon_{re}}/(cZ_c) - C_p$$

    - $C_f'$ accounts for the modification of fringe capacitance $C_f$:

      $$C_f' = \frac{C_f}{1 + A(h/s)\tanh(8s/h)} \quad , \quad A = \exp[-0.1\,\exp(2.33 - 2.53W/h)]$$

    - $C_{gd}$ may be found from the corresponding coupled stripline geometry:

      $$C_{gd} = \frac{\varepsilon_o \varepsilon_r}{\pi} \ln\left[\coth\left(\frac{\pi}{4}\frac{s}{h}\right)\right] + 0.65C_f\left(\frac{0.02\sqrt{\varepsilon_r}}{s/h} + 1 - \frac{1}{\varepsilon_r^2}\right)$$

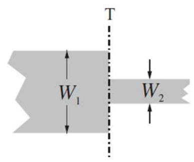
    - $C_{ga}$ can be modified from the capacitance of the corresponding coplanar strips:

      $$C_{ga} = \varepsilon_o \frac{K(k')}{K(k)} \quad , \quad \frac{K(k')}{K(k)} = \begin{cases} \dfrac{1}{\pi}\ln\left(2\dfrac{1+\sqrt{k'}}{1-\sqrt{k'}}\right) & \text{for } 0 \le k^2 \le 0.5 \\ \dfrac{\pi}{\ln\left(2\dfrac{1+\sqrt{k}}{1-\sqrt{k}}\right)} & \text{for } 0.5 \le k^2 \le 1 \end{cases} \quad , \quad k = \frac{s/h}{s/h + 2W/h}, \quad k' = \sqrt{1-k^2}$$



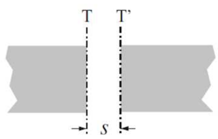
# Discontinuities And Components
# – Discontinuities

- Microstrip discontinuities commonly encountered in the layout of practical filters include steps, open-ends, bends, gaps, and junctions.
- The effects of discontinuities can be accurately modeled by full-wave EM simulator or closed-form expressions and taken into account in the filter designs.
  - Steps in width:

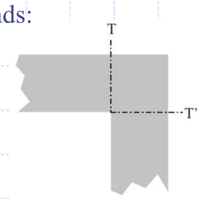



  - Open ends:

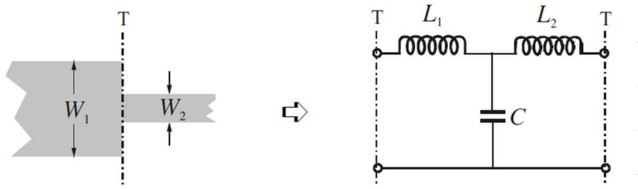

  - Gaps:




  - Bends:



# Discontinuities
# – Steps in width




$$C = 0.00137h \frac{\sqrt{\varepsilon_{re1}}}{Z_{c1}}\left(1 - \frac{W_2}{W_1}\right)\left(\frac{\varepsilon_{re1} + 0.3}{\varepsilon_{re1} - 0.258}\right)\left(\frac{W_1/h + 0.264}{W_1/h + 0.8}\right) \text{(pF)}$$

$$L_1 = \frac{L_{w1}}{L_{w1} + L_{w2}} L \qquad L_{wi} = Z_{ci}\sqrt{\varepsilon_{rei}}/c$$

where

$$L_2 = \frac{L_{w2}}{L_{w1} + L_{w2}} L \qquad L = 0.000987h\left(1 - \frac{Z_{c1}}{Z_{c2}}\sqrt{\frac{\varepsilon_{re1}}{\varepsilon_{re2}}}\right)^2$$

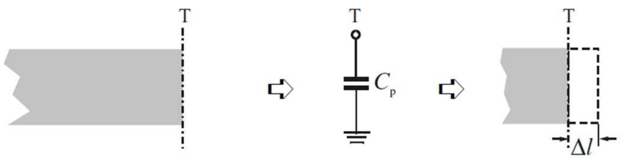
Note : $L_{wi}$ for i = 1, 2 are the inductances per unit length of the appropriate microistrips, having widths $W_1$ and $W_2$, respectively.

$Z_{ci}$ and $\varepsilon_{rei}$ denote the characteristic impedance and effective dielectric constant corresponding to width $W_i$, and h is the substrate thickness in micrometers.



# Discontinuities
# – Open ends

- The fields do not stop abruptly but extend slightly further due to the effect of the fringing field.




$$\Delta l = \frac{cZ_cC_p}{\sqrt{\varepsilon_{re}}}$$

- Closed-form expression:

$$\frac{\Delta l}{h} = \frac{\xi_1\xi_3\xi_5}{\xi_4} \quad \text{where} \quad \xi_1 = 0.434907\frac{\varepsilon_{re}^{0.81} + 0.26(W/h)^{0.8544} + 0.236}{\varepsilon_{re}^{0.81} - 0.189(W/h)^{0.8544} + 0.87}$$

$$\xi_2 = 1 + \frac{(W/h)^{0.371}}{2.35\varepsilon_r + 1}$$

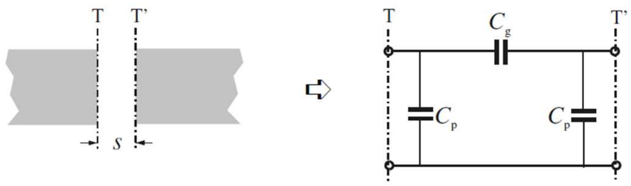
$$\xi_3 = 1 + \frac{0.5274\tan^{-1}[0.084(W/h)^{1.9413/\xi_2}]}{\varepsilon_{re}^{0.9236}}$$

$$\xi_4 = 1 + 0.037\tan^{-1}[0.067(W/h)^{1.456}]\cdot\{6 - 5\exp[0.036(1 - \varepsilon_r)]\}$$

$$\xi_5 = 1 - 0.218\exp(-7.5W/h)$$

- The accuracy is better than 0.2 % for the range of $0.01 \le W/h \le 100$ and $\varepsilon_r \le 128$

# Discontinuities – Gaps




$$C_p = 0.5C_e$$
$$C_g = 0.5C_o - 0.25C_e$$

$$\frac{C_o}{W}(\text{pF/m}) = \left(\frac{\varepsilon_r}{9.6}\right)^{0.8}\left(\frac{s}{W}\right)^{m_o}\exp(k_o)$$

$$\frac{C_e}{W}(\text{pF/m}) = 12\left(\frac{\varepsilon_r}{9.6}\right)^{0.9}\left(\frac{s}{W}\right)^{m_e}\exp(k_e)$$

where

$$m_o = \frac{W}{h}[0.619\log(W/h) - 0.3853] \qquad \text{for } 0.1 \le s/W \le 1.0$$
$$k_o = 4.26 - 1.453\log(W/h)$$

$$m_e = 0.8675 \qquad \text{for } 0.1 \le s/W \le 0.3$$
$$k_e = 2.043\left(\frac{W}{h}\right)^{0.12}$$

$$m_e = \frac{1.565}{(W/h)^{0.16}} - 1 \qquad \text{for } 0.3 \le s/W \le 1.0$$
$$k_e = 1.97 - \frac{0.03}{W/h}$$

- The accuracy is within 7 % for $0.5 \le W/h \le 2$ and $2.5 \le \varepsilon_r \le 15$

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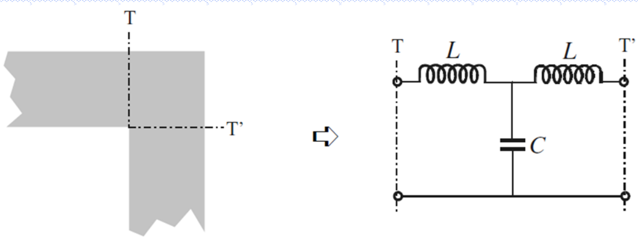
# Discontinuities – Bends




$$\frac{C}{W}(\text{pF/m}) = \begin{cases} \dfrac{(14\varepsilon_r + 12.5)W/h - (1.83\varepsilon_r - 2.25)}{\sqrt{W/h}} + \dfrac{0.02\varepsilon_r}{W/h} & \text{for } W/h < 1 \\ (9.5\varepsilon_r + 1.25)W/h + 5.2\varepsilon_r + 7.0 & \text{for } W/h \ge 1 \end{cases}$$

$$\frac{L}{h}(\text{nH/m}) = 100\left\{4\sqrt{\frac{w}{h}} - 4.21\right\}$$

- The accuracy on the capacitance is quoted as within 5% over the ranges of $2.5 \le \varepsilon_r \le 15$ and $0.1 \le W/h \le 5$.
- The accuracy on the inductance is about 3 % for $0.5 \le W/h \le 2$.

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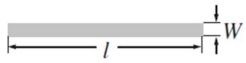
# Components
# – lumped inductors and capacitors

- Lumped inductors and capacitors

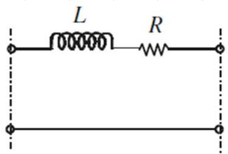
  The elements whose physical dimensions are much smaller than the free space wavelength $\lambda_0$ of the highest operating frequency (smaller than 0.1 $\lambda_0$).

- Design of inductors
  - High-impedance line

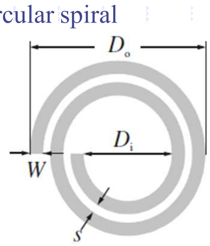



  - Meander line

  - Circuit representation




  - Circular spiral




  - Square spiral

  - Initial design formula for straight-line inductor

$$L(\text{nH}) = 2 \times 10^{-4}\, l \left[ \ln\left(\frac{l}{W+t}\right) + 1.193 + 0.2235 \frac{W+t}{l} \right] \cdot K_g \quad \text{for } l \text{ in } \mu\text{m}$$

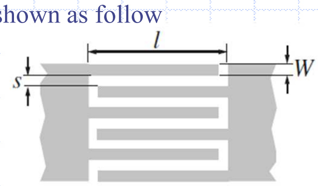
$$R = \frac{R_s l}{2(W+t)} \cdot \left[ 1.4 + 0.217 \ln\left(\frac{W}{5t}\right) \right] \quad \text{for } 5 < \frac{W}{t} < 100$$



# Components
# – lumped inductors and capacitors

- Design of capacitors
  - Interdigital capacitor

    Assuming the finger width W equals to the space and empirical formula for capacitance is shown as follow

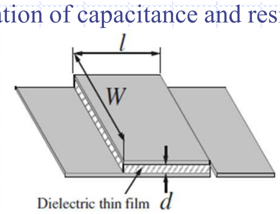



$$C(\text{pF}) = 3.937 \times 10^{-5}\, l(\varepsilon_r + 1)[0.11(n-3) + 0.252] \quad \text{for } l \text{ in } \mu\text{m}$$

$$R = \frac{4}{3} \frac{R_s l}{Wn}$$

  - Metal-insulator-metal (MIM) capacitor

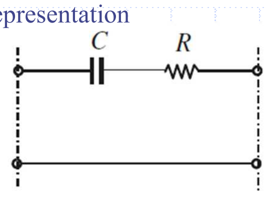
    Estimation of capacitance and resistance is approximated by parallel-plate




$$C = \frac{\varepsilon(W \times l)}{d}$$

$$R = \frac{R_s l}{W}$$

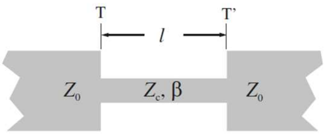
  - Circuit representation

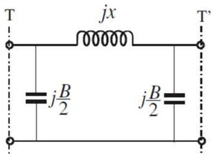


# Components
# – Quasilumped elements (1)

- Quasilumped elements
  Physical lengths are smaller than a quarter of guided wavelength $\lambda_g$.
  - High-impedance short line element

$l < \frac{\lambda_g}{8}$

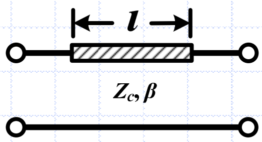


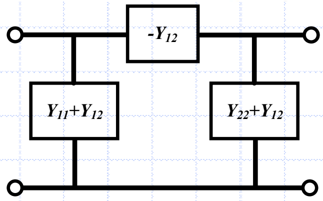




$$x = Z_c \sin\left(\frac{2\pi}{\lambda_g} l\right) \approx Z_c\left(\frac{2\pi}{\lambda_g} l\right)$$

$$\frac{B}{2} = \frac{1}{Z_c}\tan\left(\frac{\pi}{\lambda_g} l\right) \approx \frac{1}{Z_c}\left(\frac{\pi}{\lambda_g} l\right)$$

- Derivation




$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \cos\beta\ell & jZ_c\sin\beta\ell \\ j\frac{1}{Z_c}\sin\beta\ell & \cos\beta\ell \end{bmatrix} \Rightarrow \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \begin{bmatrix} \frac{D}{B} & \frac{-(AD-BC)}{B} \\ \frac{-1}{B} & \frac{A}{B} \end{bmatrix} = \begin{bmatrix} \frac{\cos\beta\ell}{jZ_c\sin\beta\ell} & \frac{-1}{jZ_c\sin\beta\ell} \\ \frac{-1}{jZ_c\sin\beta\ell} & \frac{\cos\beta\ell}{jZ_c\sin\beta\ell} \end{bmatrix}$$

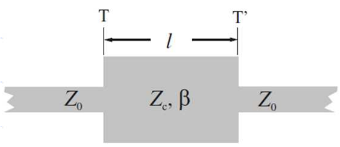
inductive element: $-Y_{12} = \frac{1}{jZ_c\sin\beta\ell} = \frac{1}{jx}$

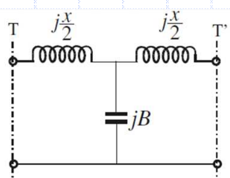
capacitive element: $Y_{11} + Y_{12} = \frac{\cos\beta\ell - 1}{jZ_c\sin\beta\ell} = \frac{\cos^2\frac{\beta\ell}{2} - \sin^2\frac{\beta\ell}{2} - \left(\cos^2\frac{\beta\ell}{2} + \sin^2\frac{\beta\ell}{2}\right)}{jZ_c 2\sin\frac{\beta\ell}{2}\cos\frac{\beta\ell}{2}} = \frac{-2\sin^2\frac{\beta\ell}{2}}{jZ_c 2\sin\frac{\beta\ell}{2}\cos\frac{\beta\ell}{2}} = j\frac{\tan\frac{\beta\ell}{2}}{Z_c} = j\frac{B}{2}$

# Components
# – Quasilumped elements (2)

- Quasilumped elements
  - Low-impedance short line element

$l < \frac{\lambda_g}{8}$

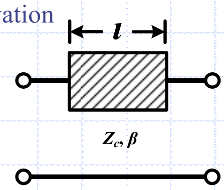


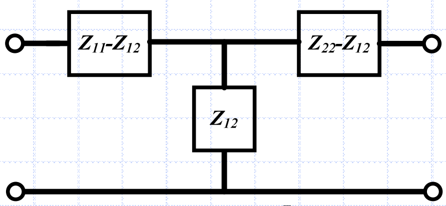




$$B = \frac{1}{Z_c}\sin\left(\frac{2\pi}{\lambda_g} l\right) \approx \frac{1}{Z_c}\left(\frac{2\pi}{\lambda_g} l\right)$$

$$\frac{x}{2} = Z_c \tan\left(\frac{\pi}{\lambda_g} l\right) \approx Z_c\left(\frac{\pi}{\lambda_g} l\right)$$

- Derivation



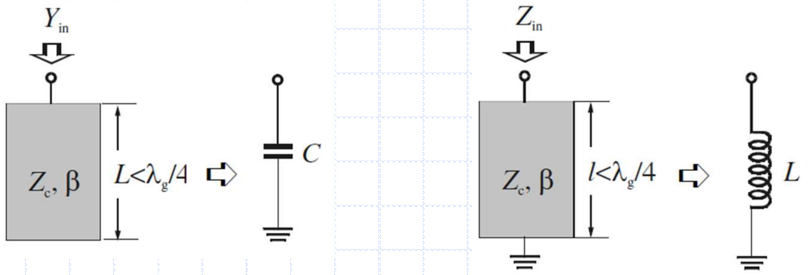

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \cos\beta\ell & jZ_c\sin\beta\ell \\ j\frac{1}{Z_c}\sin\beta\ell & \cos\beta\ell \end{bmatrix} \Rightarrow \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \begin{bmatrix} \frac{A}{C} & \frac{(AD-BC)}{C} \\ \frac{1}{C} & \frac{D}{C} \end{bmatrix} = \begin{bmatrix} \frac{Z_c\cos\beta\ell}{j\sin\beta\ell} & \frac{Z_c}{j\sin\beta\ell} \\ \frac{Z_c}{j\sin\beta\ell} & \frac{Z_c\cos\beta\ell}{j\sin\beta\ell} \end{bmatrix}$$

capacitive element: $Z_{12} = \frac{Z_c}{j\sin\beta\ell} = \frac{1}{jB} \Rightarrow B = \frac{\sin\beta\ell}{Z_c}$

inductive element: $Z_{11} - Z_{12} = \frac{Z_c}{j}\frac{\cos\beta\ell - 1}{\sin\beta\ell} = jZ_c\tan\frac{\beta\ell}{2} = j\frac{x}{2} \Rightarrow \frac{x}{2} = Z_c\tan\frac{\beta\ell}{2}$

# Components
# – Quasilumped elements (3)

- Quasilumped elements
  - Open- and short-circuited stubs
    (assuming the length L is smaller than a quarter of guided wavelength $\lambda_g$)




$$Y_{in} = jY_c \tan\left(\frac{2\pi}{\lambda_g} l\right) \approx jY_c\left(\frac{2\pi}{\lambda_g} l\right) = j\omega\left(\frac{Y_c l}{v_p}\right) \quad (C), \qquad l < \frac{\lambda_g}{8}$$

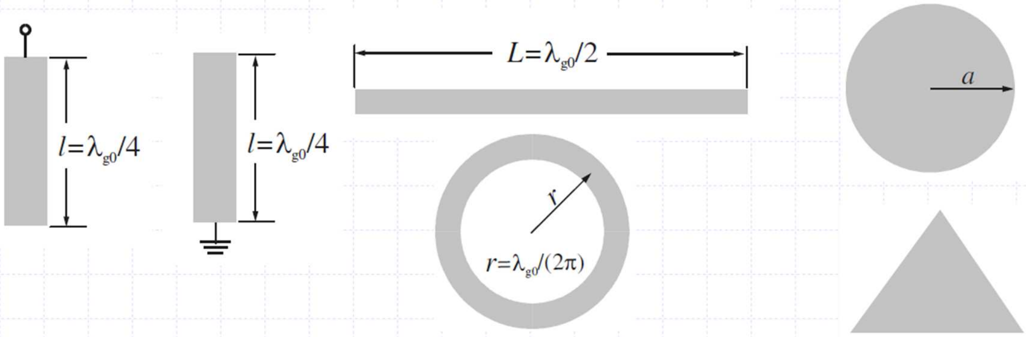
$$Z_{in} = jZ_c \tan\left(\frac{2\pi}{\lambda_g} l\right) \approx jZ_c\left(\frac{2\pi}{\lambda_g} l\right) = j\omega\left(\frac{Z_c l}{v_p}\right) \quad (L), \qquad l < \frac{\lambda_g}{8}$$



# Components
# – Resonators

- Lumped-element or quasilumped-element resonators
  - Formed by the lumped or quasilumped inductors and capacitors
  - Resonate at $\omega_0 = \frac{1}{\sqrt{LC}}$

- Distributed resonators
  - Quarter-wavelength resonators, half-wavelength resonators, and patch resonators.



# Loss Considerations for Microstrip Resonators

- Unloaded quality factor $Q_u$ is served as a justification for whether or not the required insertion loss of a bandpass filter can be met.

$$Q_u = \omega \frac{\text{Time-average energy stored in resonator}}{\text{Average power lost in resonator}}$$

- The total unloaded quality factor of a resonator can be found by adding conductor, dielectric, and radiation loss together.

$$\frac{1}{Q_u} = \frac{1}{Q_c} + \frac{1}{Q_d} + \frac{1}{Q_r}$$

EM simulator

- Quality factors $Q_c$ and $Q_d$ for a microstrip line

$$Q = \frac{\beta}{2\alpha} \Rightarrow Q_c = \frac{\pi}{\alpha_c \lambda_g}$$

$$Q_d \geq \frac{\varepsilon'}{\varepsilon''} = \frac{1}{\tan\delta} \quad \text{or} \quad Q_d = \frac{\pi}{\alpha_d \lambda_g}$$