Principles of Radiation and Antennas

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How antenna radiate: a single accelerated charged particle

- The static electric field originates at charge and is directed radially away from charge.
- At point A, the charge begins to be accelerated until reaching point B.
- The distance between the circles is that distance light would travel in time $\triangle t$, and $\triangle r = r_b - r_a = \triangle t * c$
- Charge moves slowly compared to the speed of the light, $\therefore \triangle r >> \triangle z$ and two circles are *concentric*.
- The electric field lines in the $\triangle r$ region are joined together because of required continuity of electrical lines in the absence of charges.
- This disturbance expands outward and has a transverse component E_t, which is the radiated field.
- If charges are accelerated back and forth (i.e., oscillate), a regular disturbance is created and radiation is continuous.
- This disturbance is directly analogous to a transient wave created by a stone dropped into a calm lake.

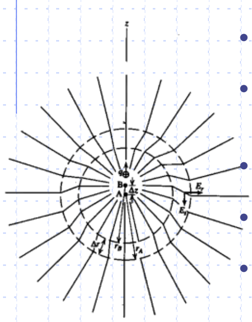
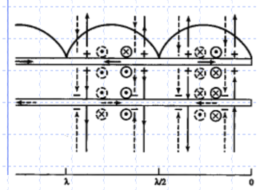
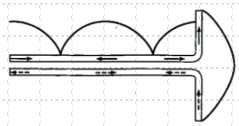


Figure 1-2 Illustration of how an accelerated charged particle radiates. Charge q moves with constant velocity in the $+z$-direction until it reaches point A (time $t = 0$), after which it accelerates to point B (time $t = \Delta t$) and then maintains its velocity. The electric field lines shown here are for a time r_B/c after the charge passed point B.

How antenna radiate : Evolution of a dipole antenna from an open-circuited transmission line



(a) Open-circuited transmission line showing currents, charges, and fields. The electric fields are indicated with lines and the magnetic fields with arrow heads and tails, solid (dashed) for those arising from the top (bottom) wire.



(b) Peak currents on a half-wavelength dipole created by bending out the ends of the transmission line.

Figure 1-3 Evolution of a dipole antenna from an open-circuited transmission line.

- **Open-circuited transmission line**

1. The currents are in opposite directions on the two wires and behaves as a standing wave pattern with a zero current magnitude at the ends and every half wavelength from the end.
2. The conductors guide the waves and the power resides in the region surrounding the conductors as manifested by the electric and magnetic fields.
3. Electric fields originate from or terminate on charges and perpendicular to the wires.
4. Magnetic fields encircle the wires.

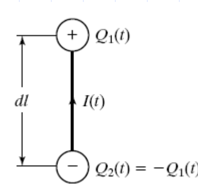
- **Bending outward to form a dipole**

1. The currents are no longer opposite but are both upwardly directed.
2. The bounded fields are exposed to the space.
3. The currents on the dipole are approximately sinusoidal.
4. The situation on the Fig. is the peak current condition. As time proceeds and current oscillation occur, the disturbed fields are radiated.

10.1 HERTZIAN DIPOLE

The *Hertzian dipole* is an elemental antenna consisting of an infinitesimally long piece of wire carrying an alternating current *I(t).*

To maintain the current flow in the wire, we postulate two point charges Q1(t) and Q2(t), terminating the wire at its two ends, so that the law of conservation of charge is satisfied.



if

$$I(t) = I_0 \cos \omega t$$

→

$$\frac{dQ_1}{dt} = I(t) = I_0 \cos \omega t$$

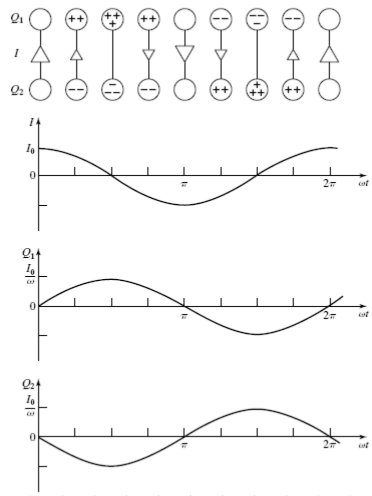
$$\frac{dQ_2}{dt} = -I(t) = -I_0 \cos \omega t$$

→

$$Q_1(t) = \frac{I_0}{\omega} \sin \omega t$$

$$Q_2(t) = -\frac{I_0}{\omega} \sin \omega t = -Q_1(t)$$

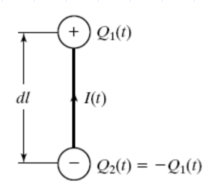
10.1 HERTZIAN DIPOLE



10.1 HERTZIAN DIPOLE

We recall from Section 5.2

$$\mathbf{A} = \frac{\mu I\, d\mathbf{l}}{4\pi r} = \frac{\mu I\, dl}{4\pi r}\mathbf{a}_z$$

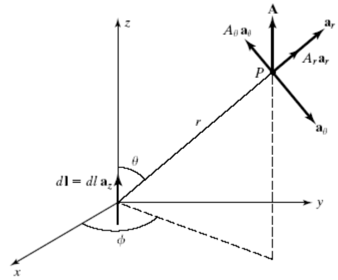


If the current in the element is now assumed to be time varying in the manner $I = I_0 \cos \omega t$, we might expect the corresponding magnetic vector potential to be that in (10.4) with I replaced by $I_0 \cos \omega t$.

The effect field at a distance r from the origin at time t is due to the current that existed at the origin at an earlier time $(t - r/v_p)$

10.1 HERTZIAN DIPOLE

$$\mathbf{A} = \frac{\mu I_0\, dl}{4\pi r} \cos \omega\left(t - \frac{r}{v_p}\right) \mathbf{a}_z = \frac{\mu I_0\, dl}{4\pi r} \cos(\omega t - \beta r)\, \mathbf{a}_z$$



The result given by (10.5) is known as the *retarded* magnetic vector potential.

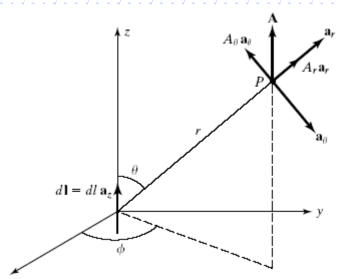
10.1 HERTZIAN DIPOLE

In terms of its components in spherical coordinates,

$$\mathbf{A} = \frac{\mu I_0\, dl \cos(\omega t - \beta r)}{4\pi r}(\cos\theta\, \mathbf{a}_r - \sin\theta\, \mathbf{a}_\theta)$$

$$\mathbf{H} = \frac{\mathbf{B}}{\mu} = \frac{1}{\mu}\nabla \times \mathbf{A}$$

$$= \frac{1}{\mu}\begin{vmatrix} \dfrac{\mathbf{a}_r}{r^2 \sin\theta} & \dfrac{\mathbf{a}_\theta}{r\sin\theta} & \dfrac{\mathbf{a}_\phi}{r} \\ \dfrac{\partial}{\partial r} & \dfrac{\partial}{\partial \theta} & \dfrac{\partial}{\partial \phi} \\ A_r & rA_\theta & 0 \end{vmatrix} = \frac{1}{\mu r}\left[\frac{\partial}{\partial r}(rA_\theta) - \frac{\partial A_r}{\partial \theta}\right]\mathbf{a}_\phi$$



$$\mathbf{H} = \frac{I_0\, dl \sin\theta}{4\pi}\left[\frac{\cos(\omega t - \beta r)}{r^2} - \frac{\beta \sin(\omega t - \beta r)}{r}\right]\mathbf{a}_\phi$$

10.1 HERTZIAN DIPOLE

$$\frac{\partial \mathbf{E}}{\partial t} = \frac{1}{\varepsilon}\nabla \times \mathbf{H}$$

$$= \frac{1}{\varepsilon}\begin{vmatrix} \frac{\mathbf{a}_r}{r^2 \sin\theta} & \frac{\mathbf{a}_\theta}{r\sin\theta} & \frac{\mathbf{a}_\phi}{r} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ 0 & 0 & r\sin\theta\, H_\phi \end{vmatrix}$$

$$= \frac{1}{\varepsilon r^2 \sin\theta}\frac{\partial}{\partial\theta}(r\sin\theta\, H_\phi)\,\mathbf{a}_r - \frac{1}{\varepsilon r\sin\theta}\frac{\partial}{\partial r}(r\sin\theta\, H_\phi)\,\mathbf{a}_\theta$$

$$\boxed{\begin{aligned}\mathbf{E} = {} & \frac{2I_0\, dl\cos\theta}{4\pi\varepsilon\omega}\left[\frac{\sin(\omega t - \beta r)}{r^3} + \frac{\beta\cos(\omega t - \beta r)}{r^2}\right]\mathbf{a}_r \\ & + \frac{I_0\, dl\sin\theta}{4\pi\varepsilon\omega}\left[\frac{\sin(\omega t - \beta r)}{r^3} + \frac{\beta\cos(\omega t - \beta r)}{r^2} - \frac{\beta^2\sin(\omega t - \beta r)}{r}\right]\mathbf{a}_\theta\end{aligned}}$$

10.1 HERTZIAN DIPOLE

They contain terms involving $1/r^3$, $1/r^2$, and $1/r$. Far from the dipole such that $\beta r \gg 1$, the $1/r^3$ and $1/r^2$ terms are negligible compared to the $1/r$ terms so that the fields vary inversely with r. Furthermore, for

Thus, the time-average Poynting vector varies proportionately to $1/r^2$ and is directed entirely in the radial direction.

10.2 RADIATION RESISTANCE AND DIRECTIVITY

Radiation fields

it is seldom necessary to work with the complete field expressions because one is often interested in the field far from the dipole that is governed predominantly by the terms involving $1/r$

$$I = I_0 \cos \omega t$$

$$\mathbf{E} = -\frac{\beta^2 I_0\, dl \sin\theta}{4\pi\varepsilon\omega r}\sin(\omega t - \beta r)\,\mathbf{a}_\theta$$
$$= -\frac{\eta\beta I_0\, dl \sin\theta}{4\pi r}\sin(\omega t - \beta r)\,\mathbf{a}_\theta$$
$$\mathbf{H} = -\frac{\beta I_0\, dl \sin\theta}{4\pi r}\sin(\omega t - \beta r)\,\mathbf{a}_\phi$$

These fields are known as the *radiation fields*, since they are the components of the total fields that contribute to the time-average radiated power away from the dipole.

Steps in evaluation of radiation fields : Solution of Maxwell equations for radiation problems)

Magnetic vector potential

Electric scalar potential

$$\nabla\cdot\vec{H} = 0 \longrightarrow \vec{H} = \frac{1}{\mu}\nabla\times\vec{A}$$

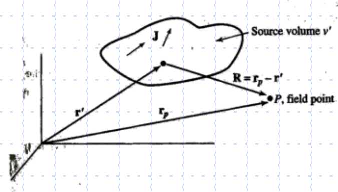
$$\nabla\times\vec{E} = -j\omega\mu\vec{H} \longrightarrow \nabla\times(\vec{E}+j\omega\vec{A}) = 0 \longrightarrow \vec{E}+j\omega\vec{A} = -\nabla\phi$$

$$\nabla\times\vec{H} = j\omega\varepsilon\vec{E}+\vec{J} \longrightarrow \nabla\times\nabla\times\vec{A} = \nabla(\nabla\cdot\vec{A})-\nabla^2\vec{A} = j\omega\mu\varepsilon(-j\omega\vec{A}-\nabla\phi)+\mu\vec{J}$$

$$\nabla\cdot\vec{E} = \frac{\rho}{\varepsilon}$$

If we set $\nabla\cdot\vec{A} = -j\omega\mu\varepsilon\phi$ (Lorenz condition)

Vector wave equation $\longrightarrow$ $$\nabla^2\vec{A}+\omega^2\mu\varepsilon\vec{A} = -\mu\vec{J}$$



Solution is

$$\vec{A} = \iiint_{v'} \mu\vec{J}\,\frac{e^{-j\beta R}}{4\pi R}\,dv'$$

Figure 1-8 Vectors used to solve radiation problems.

Steps in evaluation of radiation fields (Far Fields)

1. Find **A**

Far field
$$\vec{A} = \mu \frac{e^{-j\beta r}}{4\pi r} \iiint_{v'} \vec{J} e^{j\beta \hat{r}\cdot r'} dv'$$

Z-directed sources
$$\vec{A} = \hat{z}\mu \frac{e^{-j\beta r}}{4\pi r} \iiint_{v'} J_z e^{j\beta \hat{r}\cdot r'} dv'$$

Z-directed line sources
$$\vec{A} = \hat{z}\mu \frac{e^{-j\beta r}}{4\pi r} \iiint_{v'} I(z') e^{j\beta z' \cdot \cos\theta} dv'$$

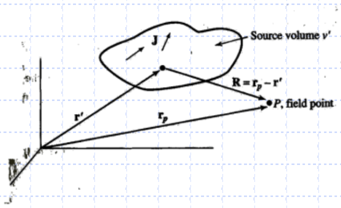


Figure 1-8 Vectors used to solve radiation problems.

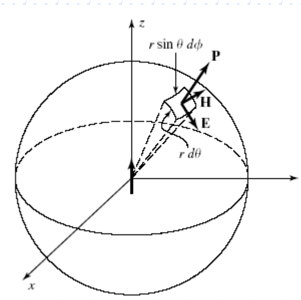
10.2 RADIATION RESISTANCE AND DIRECTIVITY

Radiation fields

The Poynting vector due to the radiation fields is given by

$$\begin{aligned}\mathbf{P} &= \mathbf{E} \times \mathbf{H} \\ &= E_\theta \mathbf{a}_\theta \times H_\phi \mathbf{a}_\phi = E_\theta H_\phi \mathbf{a}_r \\ &= \frac{\eta\beta^2 I_0^2 (dl)^2 \sin^2\theta}{16\pi^2 r^2} \sin^2(\omega t - \beta r)\, \mathbf{a}_r\end{aligned}$$

$$\begin{aligned}P_{\text{rad}} &= \int_{\theta=0}^{\pi}\int_{\phi=0}^{2\pi} \mathbf{P}\cdot r^2 \sin\theta\, d\theta\, d\phi\, \mathbf{a}_r \\ &= \int_{\theta=0}^{\pi}\int_{\phi=0}^{2\pi} \frac{\eta\beta^2 I_0^2 (dl)^2 \sin^3\theta}{16\pi^2} \sin^2(\omega t - \beta r)\, d\theta\, d\phi \\ &= \frac{\eta\beta^2 I_0^2 (dl)^2}{8\pi} \sin^2(\omega t - \beta r) \int_{\theta=0}^{\pi} \sin^3\theta\, d\theta \\ &= \frac{\eta\beta^2 I_0^2 (dl)^2}{6\pi} \sin^2(\omega t - \beta r) \\ &= \frac{2\pi\eta I_0^2}{3}\left(\frac{dl}{\lambda}\right)^2 \sin^2(\omega t - \beta r)\end{aligned}$$



10.2 RADIATION RESISTANCE AND DIRECTIVITY

Radiation fields

The time-average power radiated by the dipole, that is, the average of P_{rad} over one period of the current variation, is

$$\begin{aligned}\langle P_{rad}\rangle &= \frac{2\pi\eta I_0^2}{3}\left(\frac{dl}{\lambda}\right)^2\langle\sin^2(\omega t-\beta r)\rangle \\ &= \frac{\pi\eta I_0^2}{3}\left(\frac{dl}{\lambda}\right)^2 \qquad (10.20) \\ &= \frac{1}{2}I_0^2\left[\frac{2\pi\eta}{3}\left(\frac{dl}{\lambda}\right)^2\right]\end{aligned}$$

10.2 RADIATION RESISTANCE AND DIRECTIVITY

Radiation Resistance

Re- calling that the average power dissipated in a resistor R when a current $I_0 \cos \omega t$ is passed through it is $\frac{1}{2}I_0^2R$, we note from (10.20) that the radiation re-

$$R_{rad} = \frac{2\pi\eta}{3}\left(\frac{dl}{\lambda}\right)^2 \Omega$$

For free space, $\eta = \eta_0 = 120\pi\ \Omega$, and

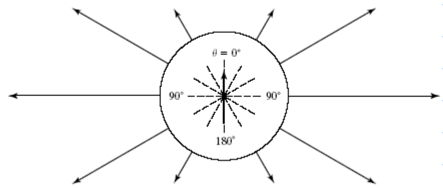
$$R_{rad} = 80\pi^2\left(\frac{dl}{\lambda}\right)^2 \Omega$$

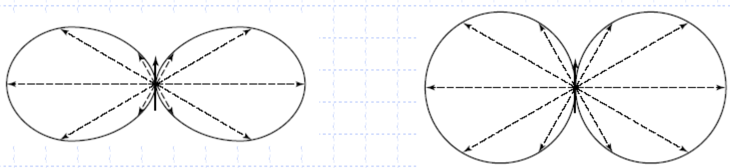
As a numerical example, for (dl/λ) equal to 0.01, $R_{rad} = 80\pi^2(0.01)^2 = 0.08\ \Omega$. Thus, for a current of peak amplitude 1 A, the time-average radiated power is equal to 0.04 W. This indicates that a Hertzian dipole of length 0.01λ is not a very effective radiator.

10.2 RADIATION RESISTANCE AND DIRECTIVITY

Radiation Pattern

We note from (10.17a) and (10.17b) that, for a constant *r*, the amplitude of the fields is proportional to sinθ. Similarly, we note from (10.18) that for a constant *r*, the power density is proportional to $\sin^2\theta$





Antenna Parameters: radiation pattern

The field pattern with its maximum value is 1 → $F(\theta,\phi)=\dfrac{E_\theta}{E_{\theta,\max}}$

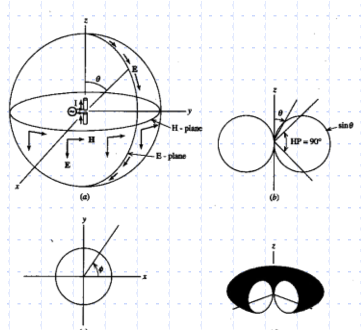


Figure 1-10 Radiation from an ideal dipole. (*a*) Field components. (*b*) *E*-plane radiation pattern polar plot of $|E_\theta|$ or $|H_\phi|$. (*c*) *H*-plane radiation pattern polar plot of $|E_\theta|$ or $|H_\phi|$. (*d*) Three-dimensional plot of radiation pattern.

For Hertzian dipole

F(θ)=sinθ

10.2 RADIATION RESISTANCE AND DIRECTIVITY

Directivity

We now define a parameter known as the *directivity* of the antenna, denoted by the symbol *D*, as the ratio of the maximum power density radiated by the antenna to the average power density.

$$[P_r]_{max} = \frac{\eta\beta^2 I_0^2 (dl)^2 [\sin^2\theta]_{max}}{16\pi^2 r^2} \sin^2(\omega t - \beta r)$$
$$= \frac{\eta\beta^2 I_0^2 (dl)^2}{16\pi^2 r^2} \sin^2(\omega t - \beta r)$$

$$[P_r]_{av} = \frac{P_{rad}}{4\pi r^2} = \frac{\eta\beta^2 I_0^2 (dl)^2}{24\pi^2 r^2} \sin^2(\omega t - \beta r)$$

$$\boxed{D = \frac{[P_r]_{max}}{[P_r]_{av}} = 1.5}$$

10.2 RADIATION RESISTANCE AND DIRECTIVITY

Radiation Pattern

To generalize the computation of directivity for an arbitrary radiation pattern, let us consider

$$P_r = \frac{P_0 \sin^2(\omega t - \beta r)}{r^2} f(\theta, \phi)$$

where P_0 is a constant, and $f(\theta, \phi)$ is the power density pattern.

$$[P_r]_{max} = \frac{P_0 \sin^2(\omega t - \beta r)}{r^2} [f(\theta, \phi)]_{max}$$

$$[P_r]_{av} = \frac{P_{rad}}{4\pi r^2}$$
$$= \frac{1}{4\pi r^2} \int_{\theta=0}^{2\pi} \int_{\phi=0}^{\pi} \frac{P_0 \sin^2(\omega t - \beta r)}{r^2} f(\theta, \phi)\, \mathbf{a}_r \cdot r^2 \sin\theta\, d\theta\, d\phi\, \mathbf{a}_r$$
$$= \frac{P_0 \sin^2(\omega t - \beta r)}{4\pi r^2} \int_{\theta=0}^{\pi} \int_{\phi=0}^{2\pi} f(\theta, \phi) \sin\theta\, d\theta\, d\phi$$

$$\boxed{D = 4\pi \frac{[f(\theta, \phi)]_{max}}{\int_{\theta=0}^{\pi} \int_{\phi=0}^{2\pi} f(\theta, \phi) \sin\theta\, d\theta\, d\phi}} \qquad (10.27)$$